A STUDY OF METHODS USED IN

MEASUREMENT AND ANALYSIS OF SEDIMENT LOADS IN STREAMS

REPORT NO. 10

ACCURACY OF SEDIMENT SIZE ANALYSES MADE BY THE BOTTOM WITHDRAWAL TUBE METHOD

APRIL 1953

A Study of Methods Used in

MEASUREMENT AND ANALYSIS OF SEDIMENT LOADS IN STREAMS

A Cooperative Project

Sponsored by the

Subcommittee on Sedimentation
Federal Inter-Agency River Basin Committee

Participating Agencies

Tennessee Valley Authority
Corps of Engineers ** Geological Survey
Soil Conservation Service ** Coast and Geodetic Survey
Bureau of Reclamation ** Forest Service
Federal Power Commission

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The cooperative study of methods used in

MEASUREMENT AND ANALYSIS OF SEDIMENT LOADS IN STREAMS

covers phases indicated by the following report titles.

Report No. 1

FIELD PRACTICE AND EQUIPMENT USED IN SAMPLING SUSPENDED SEDIMENT

Report No. 2

EQUIPMENT USED FOR SAMPLING BED-LOAD AND BED MATERIAL

Report No. 3

ANALYTICAL STUDY OF METHODS OF SAMPLING SUSPENDED SEDIMENT

Report No. 4

METHODS OF ANALYZING SEDIMENT SAMPLES

Report No. 5

LABORATORY INVESTIGATIONS OF SUSPENDED SEDIMENT SAMPLERS

Report No. 6

THE DESIGN OF IMPROVED TYPES OF SUSPENDED SEDIMENT SAMPLERS

Report No. 7

A STUDY OF NEW METHODS FOR SIZE ANALYSIS OF SUSPENDED SEDIMENT SAMPLES

Report No. 8

MEASUREMENT OF THE SEDIMENT DISCHARGE OF STREAMS

Report No. 9

DENSITY OF SEDIMENTS DEPOSITED IN RESERVOIRS

Report No. 10

ACCURACY OF SEDIMENT SIZE ANALYSES MADE BY THE BOTTOM WITHDRAWAL TUBE METHOD

Miscellaneous and progress reports titled as follows have been prepared:

PRELIMINARY FIELD TESTS OF THE U.S. SEDIMENT-SAMPLING EQUIPMENT
IN THE COLORADO RIVER BASIN
APRIL 1944

FIELD CONFERENCES ON SUSPENDED SEDIMENT SAMPLING
SEPTEMBER 1944

PROGRESS REPORT, COMPARATIVE FIELD TESTS ON SUSPENDED SEDIMENT
SAMPLERS
DECEMBER 1944

PROGRESS REPORT, COMPARATIVE FIELD TESTS ON SUSPENDED SEDIMENT
SAMPLERS*
AS OF JANUARY 1946

STUDY OF METHODS USED IN MEASUREMENT AND ANALYSIS OF SEDIMENT
LOADS IN STREAMS
(Paper presented at ASCE convention, Spokane, Washington)
JULY 1946

PRELIMINARY REPORT ON U.S. DH-48 (HAND) SUSPENDED SEDIMENT SAMPLER**
APRIL 1948

OPERATION AND MAINTENANCE OF U.S. P-46 SUSPENDED SEDIMENT SAMPLER
FEBRUARY 1949

PROGRESS REPORT, FIELD TESTS ON SUSPENDED SEDIMENT SAMPLERS,
COLORADO RIVER AT BRIGHT ANGEL CREEK NEAR GRAND CANYON, ARIZONA
JUNE 1950

INVESTIGATION OF INTAKE CHARACTERISTICS OF DEPTH-INTEGRATING
SUSPENDED SEDIMENT SAMPLERS AT THE DAVID TAYLOR MODEL BASIN*

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* In preparation for publication at a future date
** Very limited distribution

SYNOPSIS

The primary purpose of this investigation was to determine the accuracy of the bottom withdrawal tube method when applied to the size analysis of suspended sediments in the range of sand sizes. Tests were confined to sizes under 700 microns (0.7 mm.) because that size limit is seldom exceeded in suspended sediments.

Glass beads were used as a sedimentation material in order to provide a more precise check on this size analysis method than would be possible with a material composed of irregular shapes. The sphericity, density, and fall velocity of the beads were established.

Report No. 7 of this series, "New Methods for Size Analysis of Suspended Sediment Samples," presented the bottom withdrawal tube method of size analysis. This method was suggested for use in the analysis of sediment samples having concentrations of from 300 to 10,000 p.p.m. and for sediment sizes up to 1,000 microns (1.0 mm.). The method was developed in the years immediately preceding 1943 but, because of the national emergency existing at that time, it was not subjected to very extensive or conclusive tests for accuracy, especially for the analysis of sediments in the sand size range. The further tests reported here were necessary to more clearly evaluate the limits of application and the accuracy of this method.

Six duplicate samples of glass beads representing thirty combinations of size range and concentration were prepared, and a pair of each were analyzed for size distribution by the bottom withdrawal tube method in each of three sedimentation laboratories. The results are compared

in this report with the basic Odén curves computed from the known size distributions in the samples. The results are also presented in a form to show the consistency obtained by analyses at various concentrations and in different size ranges, and conclusions are drawn as to the accuracy and consistency of results obtained by the bottom withdrawal tube method.

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ACCURACY OF SEDIMENT SIZE ANALYSES MADE BY THE BOTTOM WITHDRAWAL TUBE METHOD

I. INTRODUCTION

1. Scope of the general study--The various phases of the sediment problem covered in the general project, "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams," of which the present study is a part, are indicated by the following titles and brief abstracts of preceding reports that have been published.

Report No. 1--"Field Practice and Equipment used in Sampling Suspended Sediment" is a detailed review of the equipment and methods used in suspended sediment sampling from the earliest known investigations to the present, with discussions of the advantages and disadvantages of the various methods and instruments used. The requirements of a sampler which would meet all field conditions satisfactorily are set forth.

Report No. 2--"Equipment Used for Sampling Bed-Load and Bed Material" deals with bed-load and bed material in a manner similar to that in which Report No. 1 covers suspended load.

Report No. 3--"Analytical Study of Methods of Sampling Suspended Sediment" covers an investigation of the accuracy of various methods of sampling suspended sediment in a vertical section of a stream based on the latest developments in the application of turbulence theories to sediment transportation.

Report No. 4--"Methods of Analyzing Sediment Samples" describes many methods developed for determining the size of small particles in sediment analyses. Detailed instructions are given for many of the common methods in use for determining particle size gradation and total concentration of sediment in samples as developed by agencies doing extensive work in these fields.

Report No. 5--"Laboratory Investigations of Suspended Sediment Samplers" describes investigations of the effects of various intake conditions on the accuracy of sediment samples and the filling characteristics of slow filling samplers under various conditions.

Report No. 6--"The Design of Improved Types of Suspended Sediment Samplers" describes the development of various types

of integrating samplers suitable for taking vertically depth-integrated samples in flowing streams and others for taking time-integrated samples at a fixed point. Details of the adopted types are given.

Report No. 7--"A Study of New Methods for Size Analysis of Suspended Sediment Samples" gives an account of a study to develop methods of size analysis suitable for the conditions usually met in suspended sediment investigations. It describes a simple form of apparatus developed and gives detailed procedures for its use.

Report No. 8--"Measurement of the Sediment Discharge of Streams" describes methods and equipment to be used in making sediment measurements under various conditions encountered in natural streams.

Report No. 9--"Density of Sediments Deposited in Reservoirs" presents data on the apparent density of sediment deposited in various existing reservoirs. The results are summarized and certain conclusions of value in engineering studies are given.

2. Purpose of the investigation--The investigation covered by this report is an attempt to evaluate the accuracy of bottom withdrawal tube size analyses of suspended sediment samples in the sand range. The accuracy of this method as applied to sand sizes has not previously been definitely established.

Report No. 7 presents the development of the bottom withdrawal tube method of analyzing suspended sediment samples for particle size. The bottom withdrawal tube method of size analysis has subsequently been in use in several Federal sediment laboratories. A careful study of this method in these laboratories indicated that the results of some analyses were questionable when sand particles were included in the samples. A review of the supporting theory showed the bottom withdrawal tube method to be analytically sound for the range of particle sizes and concentrations encountered in normal suspended sediment samples. The present investigation was therefore directed toward checking this method of analysis by the use of samples in which the weights of material of each size

range were known.

3. Authority and personnel--This investigation was conducted under the sponsorship of the Subcommittee on Sedimentation of the Federal Inter-Agency River Basin Committee by the active cooperation of the Geological Survey, Bureau of Reclamation, and Corps of Engineers. Most of the laboratory work involved was conducted at the St. Anthony Falls Hydraulic Laboratory of the University of Minnesota, at Minneapolis, Minnesota, by George M. Watts and Clyde O. Johnson. The report was prepared by Byrnon C. Colby, George M. Watts, and Clyde O. Johnson with the cooperation of Russell P. Christensen and under the general supervision of Martin E. Nelson and Paul C. Benedict who also reviewed and edited the report. Size analyses of about 60 samples by the bottom withdrawal tube method were performed by the Geological Survey Laboratory at Lincoln, Nebraska, and analyses of another 60 samples were made by the Missouri River Division Laboratory, Corps of Engineers, at Omaha, Nebraska.

4. Acknowledgments--Many suggestions and constructive criticisms have been received from E. W. Lane, W. M. Borland, and R. E. Glover, Bureau of Reclamation; C. S. Howard, Geological Survey; R. J. Pafford, Jr., and D. C. Bondurant, Corps of Engineers; and Dr. L. G. Straub, Director of the St. Anthony Falls Hydraulic Laboratory.

II. GLASS BEADS AS A MATERIAL FOR SEDIMENTATION STUDY

5. Reasons for use of glass beads--Because this study was primarily concerned with the accuracy of sedimentation methods of size analyses, some basis for directly checking the results of the sedimentation process was essential. This lead to the conclusion that spheres, for which the sizes determined by sieving, by microscope, or by sedimentation should be the same, were the only ideal shape to use. Several substances which could be obtained in spherical form were considered for use as sedimentation test material, but glass spheres appeared to be the most feasible. Minnesota Mining and Manufacturing Company of St. Paul, Minnesota, makes glass spheres of various small diameters for commercial purposes. Table 1 shows the size classification in microns (thousandths of a millimeter) reported by the manufacturer. Because these sizes covered an appropriate range and the beads were readily obtainable, it was decided to adopt them for the purposes of this investigation.

6. Sphericity of beads--A microscopic investigation of these glass beads showed that on the average approximately 5 per cent of the particles were irregular in shape; in some sizes about 7 per cent of the particles were irregular. This fact would probably not have made any significant difference in the test results, but in order to reduce the percentage of irregular particles two methods of selecting the beads were used.

In the first method, a composite sample composed of glass beads of all desired sizes was separated into sieve fractions by usual methods of sieving. Then, each sieve fraction was carefully deposited on the sieve

TABLE 1

PARTICLE SIZE RANGE OF GLASS BEADS

Manufactured by

Minnesota Mining and Manufacturing Company

Stock Bead	Equivalent Mesh Size	Diameter in Microns		
		Coarse	Control	Fine
20	400	50	24	20
19	360	60	29	24
18	320	70	36	29
17	280	80	44	36
*16	240	90	57	44
*15	220	94--76	76--38	38
14	180	107--94	94--76	76--38
13	150	132--107	107--94	94--76
12	120	168--132	132--107	107--94
11	100	240--168	168--132	132--107
10	80	330--240	240--168	168--132
9	60	416--330	330--240	240--168
8	50	545--416	416--330	330--240
7	40	660--545	545--416	416--330
6	36	775--660	660--545	545--416
5	30	955--775	775--660	660--545
4	24	1160--955	955--775	775--660
3	20	1480--1160	1160--955	955--775
2	16	1680--1410	1410--1320	1320--1160

*Particle size for 220 and 240 are almost identical according to above tabulation. 220 size was taken from screen apertures, whereas the 240 size was converted from micron size. The grades are actually close to each other, and for all practical purposes, the 220 size can be considered as an average of the coarse portion of the above range, and 240 an average of the fine portion.

through which it had just passed and was given a very light sieving by hand. The material that passed the sieve the second time was used, whereas that retained was discarded. This treatment eliminated practically all of the most erratic shapes without removing any appreciable quantity of spherical beads, so that, not only was the percentage of deformed beads reduced, but those that remained were very close to spherical in shape even if not entirely perfect. The beads used in all reported tests were selected in this manner, except when the following more precise method was applied.

When the odd shaped particles larger than 150 microns were to be more completely eliminated, a more successful, but also more laborious, method was used. The apparatus consisted of a rectangular stainless steel plate with adjustable legs at one end. The inclination of the plate could be varied to impart a suitably slow rolling velocity to beads placed on it. A trough for introducing the particles was attached centrally to the upper edge of the inclined plate, and a receptable was placed at the lower edge. Spherical particles introduced into the trough would roll straight down the plate into the receptacle, whereas irregularly shaped particles would meander or curve entirely off the plate before reaching the receptacle. The width of the plate was made approximately 50 times the diameter of the particle under test, and the length about 700 times the diameter of the particle. Three plates were constructed to facilitate treatment of beads in the various size ranges. By microscopic inspection it was found that this process reduced the number of deformed particles to less than 1 per cent. This more precise method of selection was applied to all material larger than 150 microns

which was to be used in the bottom withdrawal tube accuracy tests.

In Fig. 1 are presented photographs of the glass beads remaining in four different size ranges after these treatments.

7. Determination of size of beads--Glass beads which were to be used in any of the tests reported as a part of this investigation were first separated into sieve fractions by the standard A.S.T.M. [1]* procedure for sieving. The sieves used corresponded to the following sizes: 700, 500, 350, 246, 175, 149, 125, 88, 74, 62.5 and 44.2 microns. Most of the irregularly shaped beads were removed as explained in the preceding section. Individual beads for fall velocity determinations were selected and the diameter of each determined with the microscope. These diameters are believed to be accurate to within 3 per cent or less. The distribution of sizes was also determined for each sieve fraction.

To determine the size gradations within each sieve fraction, a sample of the beads was distributed on a glass slide, and the size of every bead enclosed in consecutive random areas of the slide was determined under the microscope and recorded. At least 100 beads were so classified for the determination of the size distribution within each sieve fraction. A second similar test was made at another time and the two compared. These results are shown in Fig. 2, pages 24 and 25. In two instances, the results of the distribution determinations did not check satisfactorily and additional determinations were made. In those cases the two tests which seemed most representative of the group have been plotted.

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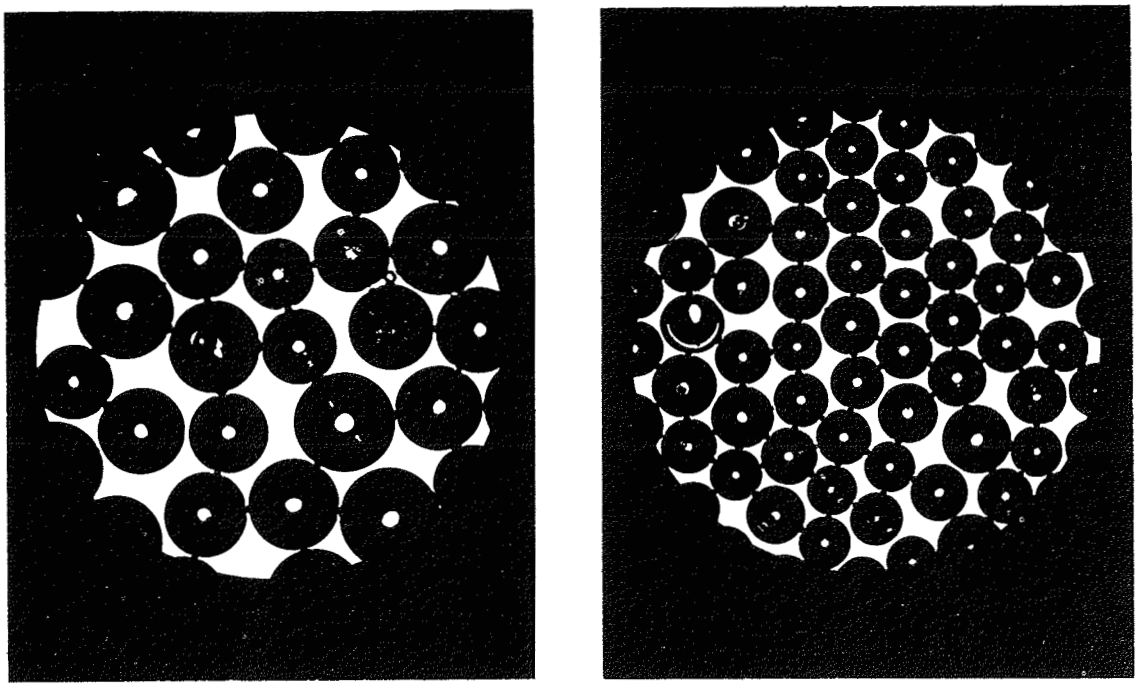
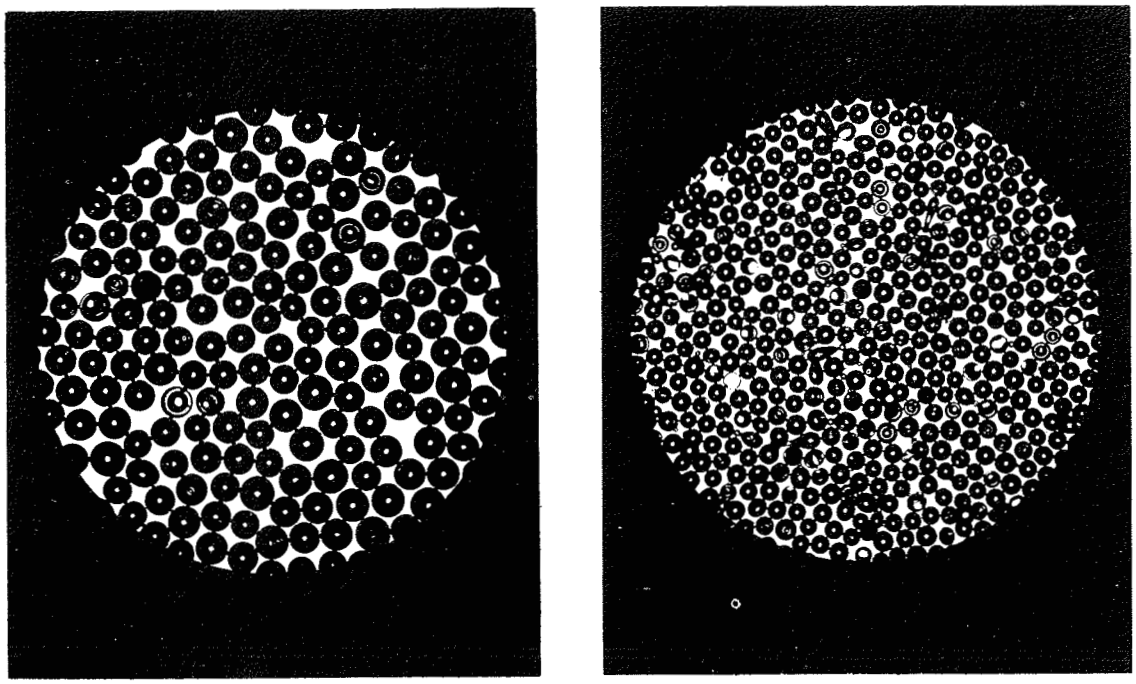
* Numbers in brackets indicate references listed in the bibliography.



500 to 700 microns - Nominal sieve sizes - 350 to 500 microns



175 to 246 microns - Nominal sieve sizes - 125 to 149 microns

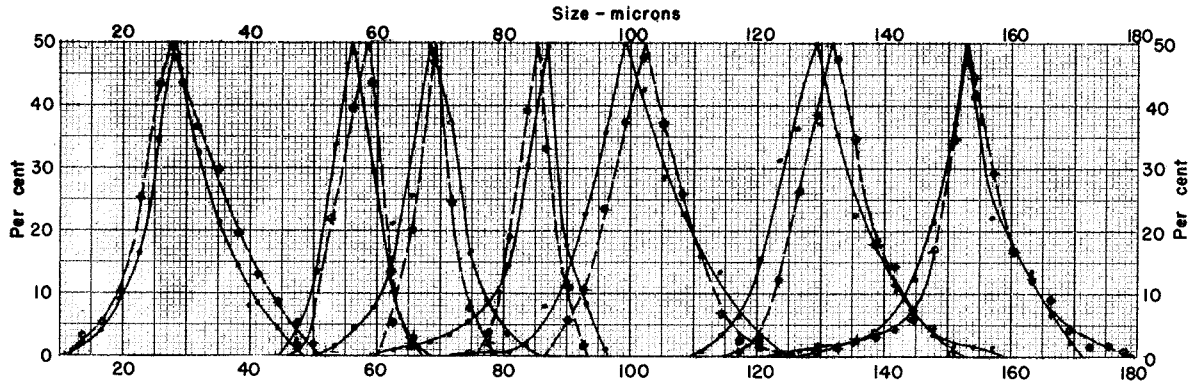
Fig. 1--Photographs of glass beads of four size ranges magnified 19 times

The first plotting of Fig. 2 shows the distribution of sizes by bead count. In this case, each sieve fraction was plotted as a unit, and the number of beads smaller than each size was expressed in per cent of the total until the percentage was 50. Then the number of beads larger than each size was plotted in per cent of the total count until no beads were larger than the size under consideration. This plotting shows the variations in the two determinations of size distribution for each sieve fraction.

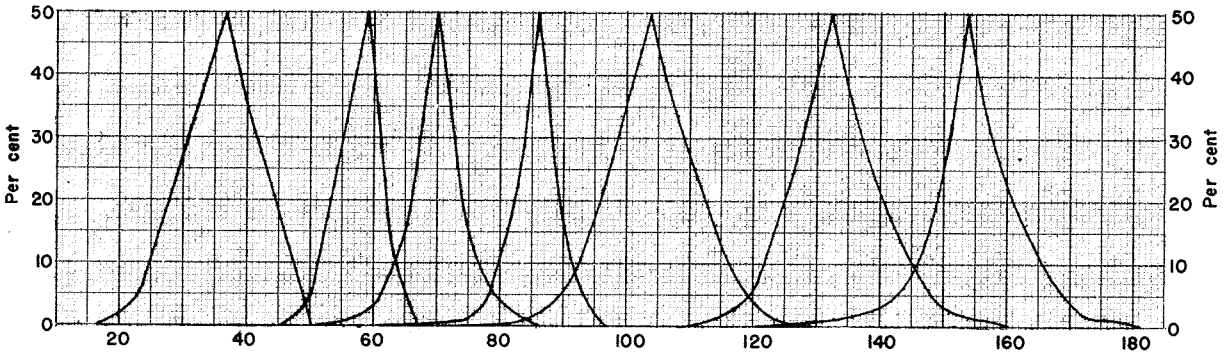
In the second method of plotting, the two determinations were combined and the total count of beads of any given size was multiplied by the cube of the diameter and the percentage was taken from these products in order to obtain the distribution of the sizes by weight. This provides a ready means for correcting the distribution of sizes in a composite sample made up of various sieve fractions, as shown by example in Section 11.

The third type of plotting presents the results of the combined size distribution determinations expressed as percentages by weight of the total sample based on the size distribution used for the test samples in the 20 to 700 micron size range. The percentage shown by each bar is that portion of the sample which is included within the size range covered by the width of the bar. The change in width scale between pages is accompanied by a compensating change in height scale so that the relative heights of the bars always represent the size frequency distribution by weight.

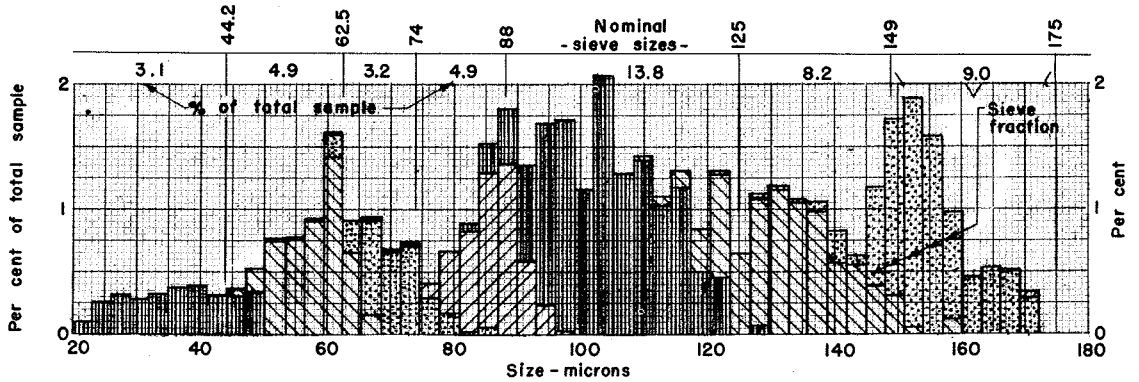
8. Density of glass beads--A preliminary investigation indicated inconsistent results in making specific gravity determinations of the



PER CENT FINER OR COARSER IN EACH SIEVE FRACTION
BASED ON BEAD COUNT
——— FIRST - DETERMINATION - SECOND - - - - -

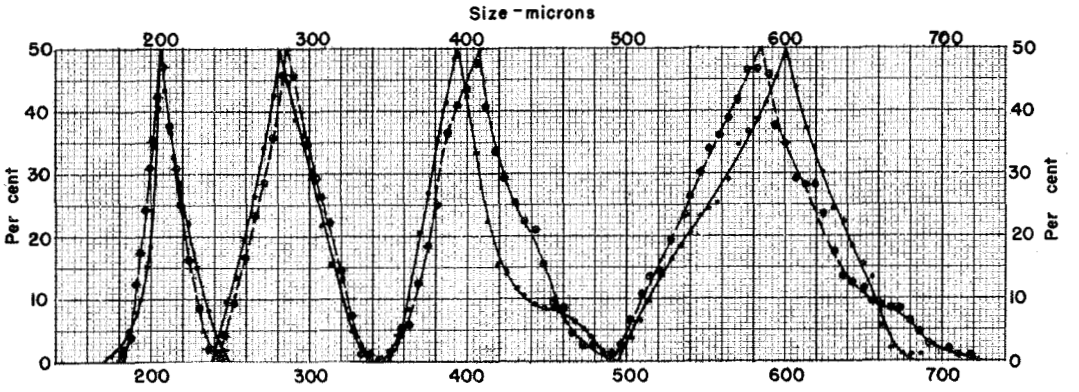


PER CENT FINER OR COARSER IN EACH SIEVE FRACTION
BY WEIGHT
TWO DETERMINATIONS COMBINED



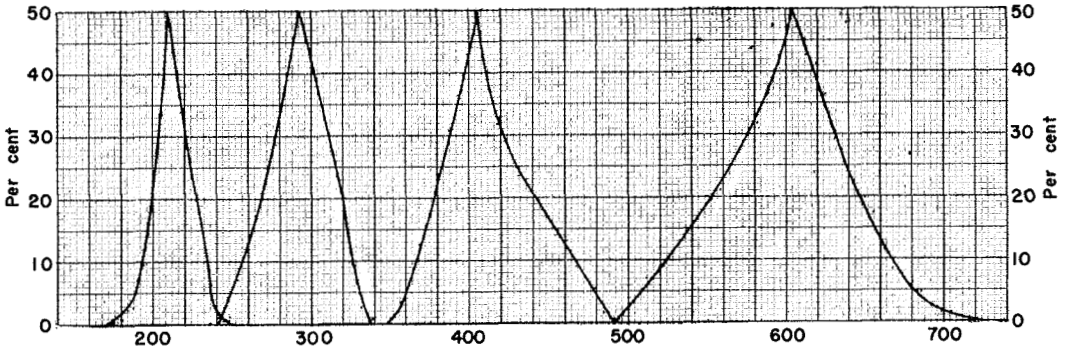
SIZE DISTRIBUTION BY WEIGHT - COMBINED DETERMINATIONS
(BASIC DISTRIBUTION USED FOR SIZE RANGE 20-700 MICRONS)

Fig. 2--Particle size distribution in glass



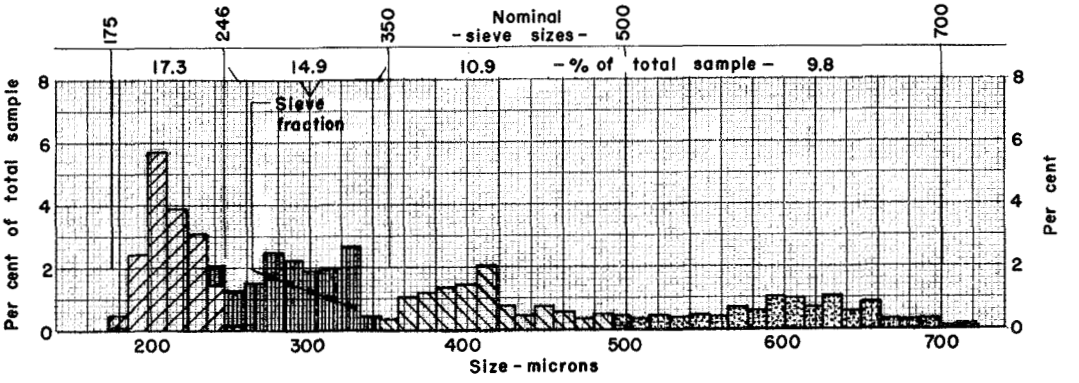
PER CENT FINER OR COARSER IN EACH SIEVE FRACTION
BASED ON BEAD COUNT

—— FIRST - DETERMINATION - SECOND -----



PER CENT FINER OR COARSER IN EACH SIEVE FRACTION
BY WEIGHT

TWO DETERMINATIONS COMBINED



SIZE DISTRIBUTION BY WEIGHT - COMBINED DETERMINATIONS
(BASIC DISTRIBUTION USED FOR SIZE RANGE 20 - 700 MICRONS)

bead samples -- microscopic determinations

stock classified glass beads; therefore, an extensive study was undertaken to establish the specific gravity, or as later expressed, the density of this material. A portion of each stock classification shown in Table 1 from No. 2 to and including No. 17 (a much lower specific gravity applies to Nos. 18, 19, and 20) was compounded into a master sample. The master sample was sieved by the standard A.S.T.M. [1] procedure for sieving. Density determinations were made of the material retained on each sieve. Determinations were run in distilled water and kerosene, using both 100 ml. pycnometers and 1,000 ml. flasks as volumetric containers. The methods and computations were based on the following outline of procedure.

1. The tare weight of the vessel to be used for the density determination was first obtained. The volume of the vessel was determined for a temperature of 20° C. by weighing the vessel full of distilled water at various temperatures, and correcting these values by means of Table 43 of the National Bureau of Standards Circular 19 [2], to give the indicated volume at 20° C.

2. The volume of the vessel at the temperature of any density determination could then be computed by use of Table 36 of the same circular. The correction was based on a cubical coefficient of expansion of glass of 0.000025 per degree Centigrade.

3. The weight per unit volume of water for the temperature of the determination could be taken from Table 37 of Circular 19. The weight per unit volume of kerosene was obtained by a series of weighings of known volumes of the liquid at a series of different temperatures.

4. The weight of the fluid, either water or kerosene, which the vessel should hold at the temperature of the density determination was found by multiplying the volume by the weight per unit volume for the given temperature.

5. The weight of the sample in a thoroughly dry condition was obtained.

6. The weight of the vessel containing the sample plus sufficient fluid to fill the vessel was determined by weighing.

7. From the sum of the weights of the dry sample, the container, and the amount of fluid needed to fill the vessel, was subtracted the weight of the vessel containing the sample plus fluid to fill the remainder of the vessel. The difference was then the weight of the fluid displaced by the sample.

8. The weight of the displaced fluid divided by the weight per unit volume of the fluid then gave the volume of fluid displaced.

9. The mass of the sample (weight corrected for air flotation), divided by the volume of displaced fluid gave the density or mass per unit volume of the sample.

The density values obtained in the final series of determinations are shown in Fig. 3. These show some variations in the individual de-

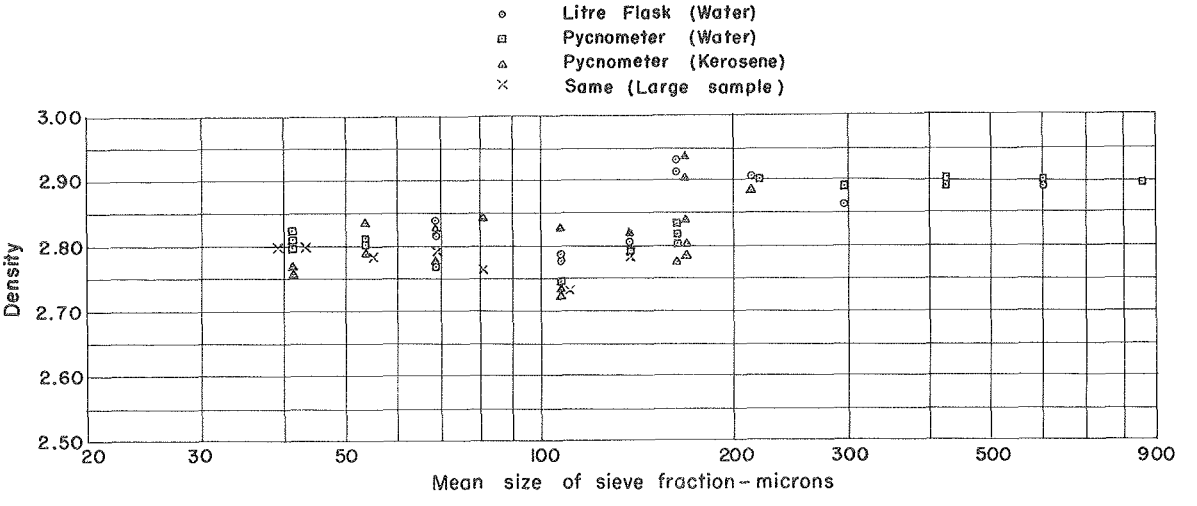


Fig. 3--Density of glass beads by sieve fractions

terminations. In addition to the errors involved in weighing and in the maintenance of uniform volumes in the containers, other factors may exert an influence as follows:

1. Incomplete wetting of the solids when the tests were made.

2. Bubbles within the glass beads.

3. Variations in density because of different rates of cooling of the beads during manufacture.

There appear to be two different densities applicable to the glass beads in the size ranges investigated. For sizes over 175 microns the density seems to be 2.89 grams per milliliter, while for sizes from 149 down to less than 44 microns the average density is about 2.80. For the size range from 149 to 175 microns the density determinations show larger differences. Probably some of the material contained in this size range has a density of about 2.89 and some a density of about 2.80.

The density of the glass beads was checked by the use of tetrabromoethane, a heavy liquid with a density of 2.950 at 20° C. The density of this fluid was cut progressively by the use of carbon tetrachloride and the action of a sample of the beads that had been mechanically dispersed in the fluid was observed at each of the resulting fluid densities. The density of the fluid was corrected for changes in temperature. The results are tabulated in Table 2. The observations shown are subject to some error in the relative quantities of beads that sank and those that floated. However, a study of the table indicates that for the larger sizes of beads a density of either 2.88 or 2.89 probably applies; and for the intermediate and smaller sizes a density of 2.82 or 2.83 would be applicable. The densities determined in tetrabromoethane are considered a fairly good check on those of Fig. 3.

9. Fall velocity of glass beads--From the relation of Reynolds number and the resistance or drag coefficient, it is possible to determine the velocity of fall of the glass beads by use of the equations in Section 14 of Report No. 4 or with the nomogram prepared by Dr. H. Rouse [3]. However, because of the scale to which these relations are drawn, the accuracy of such computations would not be very precise, and the

TABLE 2

DENSITY OF GLASS BEADS AS INDICATED BY TETRABROMOETHANE

Fluid Density	Beads Floated	Beads Sank	Remarks
	A. Size Range 175 to 1000 Microns		
2.922	all	none	
2.895	1/2-	1/2-	Some in suspension within fluid
2.858	1/5	4/5	
2.822	1/6-	5/6	Very few in suspension within fluid
2.802	none	all	
	B. Size Range 149 to 175 Microns		
2.915		none	Very few in suspension within fluid
2.864	3/5-	2/5-	Some in suspension within fluid
2.838	1/2-	1/2+	
2.815	1/3	2/3	
2.787	few	most	Some beads in a ring on meniscus
2.762	few	most	Small ring of beads on meniscus
2.687	few	most	Very small ring of beads on meniscus
	C. Size Range about 20 to 149 Microns		
2.892	all	none	
2.865	4/5-	1/5-	Some in suspension within fluid
2.828	1/3	2/3	
2.820	1/6	5/6	
2.786	1/6	5/6	
2.771	1/6	5/6	
2.744	1/6	5/6	
2.725	few	most	Very small ring of beads on meniscus

velocity would also depend upon the density assigned to the glass beads. For these reasons and also to check the relationship at moderate Reynolds numbers, it was decided to determine the fall velocity of individual glass beads.

To determine the fall velocity of individual beads, the particles were first selected and the diameters determined microscopically, and then each bead was dropped in a vertical tube in which distilled water was held at a practically constant temperature of 22° C. A few of these tests were made at other temperatures, but the results were then corrected to the base temperature, 22° C. The tube used for these fall velocity investigations was transparent, over 200 cm. in length, and 2.5 cm. in internal diameter. The individual spheres were released at the center of the water surface in the tube and a stop watch was started as the sphere fell past a point 15 cm. below the water surface. The watch was stopped when the bead reached the end of a measured distance which was never less than 10 cm. from the bottom of the tube. A fall distance of 200 cm. was used for bead sizes of 350 to 850 microns; 90 cm. for 150 to 350 microns; and 22.75 cm. or sometimes slightly less for sizes under 150 microns. A shorter fall distance was used for the smaller beads because of the difficulty of adequately lighting the tube to enable the observer to track the small beads accurately. The ratio of the diameter of the tube to the diameter of the largest particle (25 to 1) eliminated the possibility of any serious wall effects [4], and the method insured that the particle had reached its terminal fall velocity before the watch was started [5]. The fall velocities obtained experimentally are plotted in Fig. 4. Experimental fall velocities were not obtained for

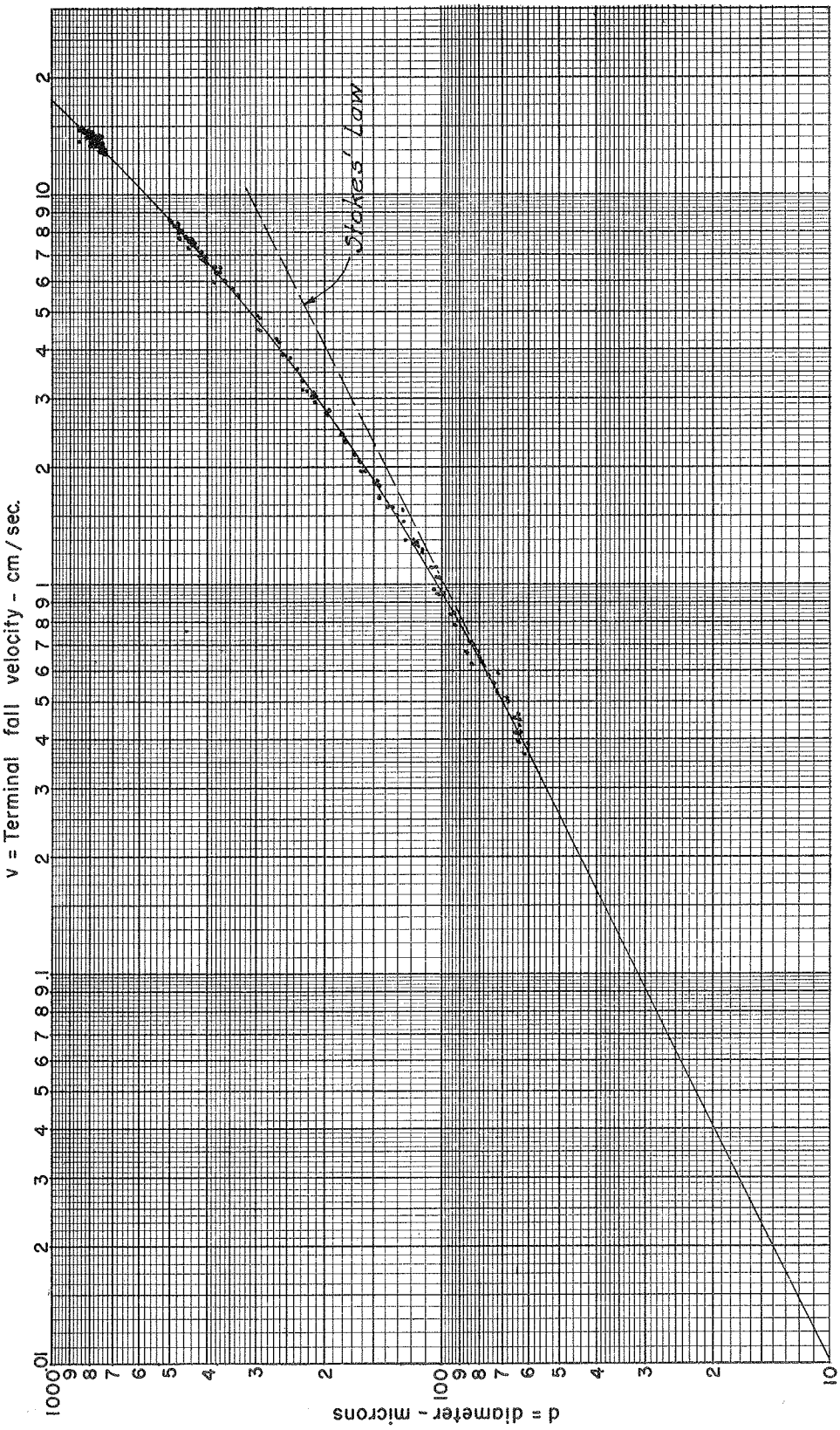


Fig. 4--Terminal fall velocity for glass spheres in distilled water at 22° C. based on experimental data

particles smaller than about 60 microns as this would require more elaborate equipment than was available. The curve of fall velocities was extended downward on the basis of Stokes' equation

$$v = \frac{2}{9} (\rho_1 - \rho_2) \frac{gr^2}{\mu} \quad \ldots\ldots\ldots\ldots \quad 1$$

where v = velocity of fall, cm./sec.

ρ_1 = 2.80, the density assigned to the glass beads in the Stokes' law size range, gm./cm.^{3}

ρ_2 = density of distilled water, gm./cm.^{3}

g = acceleration due to gravity (981 cm./sec./sec.)

r = radius of spherical particle, cm.

μ = dynamic (absolute) viscosity of water, dyne-sec./cm.^{2}

Data from the fall velocity curve of Fig. 4 were used to compute the values plotted in Fig. 5 which shows the relation between Reynolds number and the resistance coefficient as determined from the experimental data on the glass beads.

In this respect

$$R_e = \frac{vd\rho_2}{\mu} \quad \ldots\ldots\ldots \text{Reynolds number.} \ldots\ldots\ldots \quad 2$$

$$C_D = \frac{4}{3} \frac{(\rho_1 - \rho_2)}{\rho_2} \frac{gd}{v^2} \quad \ldots \text{resistance coefficient} \ldots\ldots \quad 3$$

where d = particle diameter in cm., the other symbols remaining the same as above.

Fig. 5 is identical with the relation for spheres as presented in the two references cited at the beginning of this section, that is, identical to the degree of accuracy with which it is possible to read the values from the relation for spheres as presented in these references.

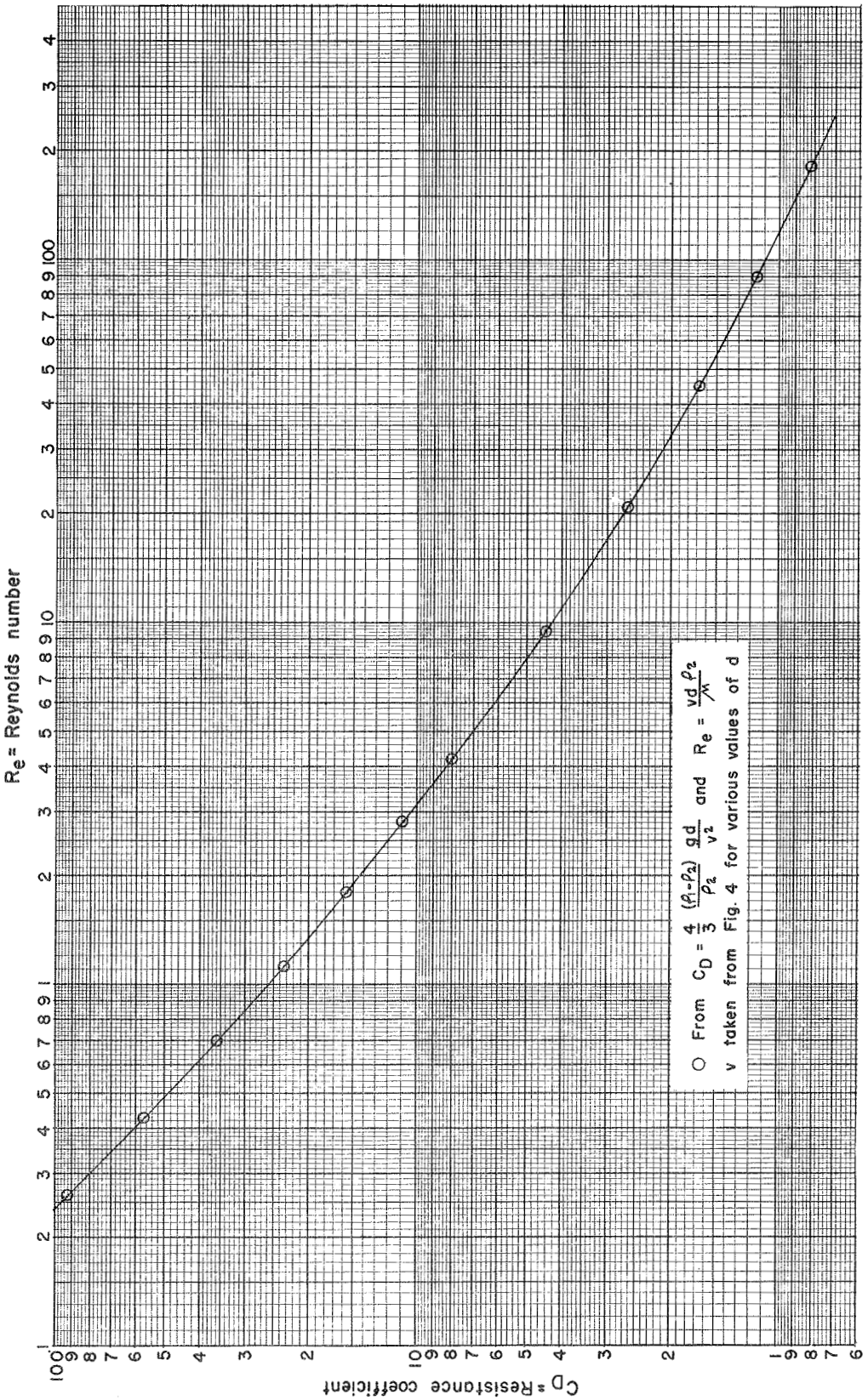


Fig. 5--Relation of Reynolds number and resistance coefficient based on experimental data for glass spheres

This also means that if the velocities from Fig. 4 had been used with the general relation for spheres to compute the density of the glass beads, the values found would have been almost exactly those assigned to the glass beads on the basis of data in Fig. 3. These relationships, together with the fact that the fall velocity curve of Fig. 4 extended smoothly into the curve defined by Stokes' equation, substantiate the experimental fall velocities obtained, and also support the density determinations made on the glass beads.

10. Time of fall table--In order to make the fall velocity data on the glass beads readily available for use in size analysis procedures, Table 3 has been computed showing the time for selected sizes of glass spheres to fall 100 cm. in water at temperatures ranging from 16° to 35° C. The sizes chosen for the table are generally those corresponding to the sieve sizes used in the tests reported in later sections, but some extra sizes have also been included.

The time of fall values for 22° have been taken directly from the fall velocity curve of Fig. 4. The time of fall values for sizes less than 62.5 microns were computed by Stokes' equation. Time of fall values for 62.5 microns and larger sizes were based on the curve of Fig. 6 which shows the relation [6] between the resistance coefficient and the quantity $F\ \rho_2/\mu^2$

where $$F = \frac{4}{3}(\rho_1 - \rho_2)\ gr^3 \quad \ldots\ldots\ldots\ldots\ 4$$

This curve was defined by using data from Fig. 4. Then $F\ \rho_2/\mu^2$ was evaluated for other temperatures, and the resistance coefficient determined from Fig. 6. The fall velocity and time of fall were computed by use of equation 3.

TABLE 3

TIME IN MINUTES FOR GLASS SPHERES TO FALL 100 CM. IN WATER

(Based on experimental determination of fall velocity of individual glass beads at 22°C)

Temp.	Particle Size in Microns													
°C.	1000	700	500	350	246	175	149	125	88	74	62.5	44.2	31.2	22.1
16	0.1002	.143	.205	.310	.486	.786	1.012	1.33	2.50	3.47	4.84	9.66	19.38	38.63
17	.0994	.142	.203	.306	.479	.774	.996	1.31	2.44	3.38	4.71	9.42	18.90	37.65
18	.0986	.140	.201	.303	.473	.762	.981	1.29	2.39	3.30	4.59	9.18	18.42	36.70
19	.0979	.139	.199	.300	.466	.751	.966	1.27	2.34	3.22	4.48	8.95	17.96	35.79
20	.0972	.138	.197	.296	.460	.739	.951	1.25	2.29	3.14	4.37	8.73	17.52	34.92
21	.0965	.137	.195	.293	.454	.728	.936	1.23	2.24	3.07	4.27	8.52	17.10	34.08
22	.0958	.136	.193	.290	.448	.718	.922	1.21	2.19	3.00	4.17	8.32	16.70	33.28
23	.0951	.134	.192	.286	.442	.708	.908	1.19	2.15	2.94	4.08	8.13	16.31	32.50
24	.0944	.133	.190	.283	.437	.698	.894	1.17	2.10	2.87	3.99	7.94	15.93	31.76
25	.0937	.132	.188	.280	.432	.689	.881	1.15	2.06	2.81	3.90	7.76	15.57	31.04
26	.0931	.131	.186	.277	.427	.680	.868	1.13	2.02	2.75	3.81	7.59	15.22	30.34
27	.0925	.130	.185	.274	.422	.671	.855	1.12	1.98	2.70	3.73	7.42	14.88	29.66
28	.0919	.129	.183	.272	.417	.662	.843	1.10	1.94	2.65	3.65	7.26	14.55	29.00
29	.0913	.128	.182	.269	.412	.653	.831	1.09	1.90	2.59	3.57	7.10	14.23	28.38
30	.0907	.127	.180	.267	.407	.644	.820	1.07	1.86	2.54	3.50	6.95	13.93	27.78
31	.0901	.126	.179	.264	.403	.636	.809	1.06	1.83	2.49	3.43	6.80	13.64	27.20
32	.0896	.125	.178	.262	.399	.628	.799	1.04	1.80	2.45	3.36	6.66	13.36	26.64
33	.0890	.125	.176	.260	.395	.620	.789	1.03	1.77	2.41	3.30	6.52	13.09	26.10
34	.0885	.124	.175	.257	.391	.612	.779	1.01	1.74	2.37	3.24	6.39	12.83	25.57
35	.0880	.123	.174	.255	.387	.605	.770	1.00	1.71	2.33	3.18	6.26	12.57	25.05

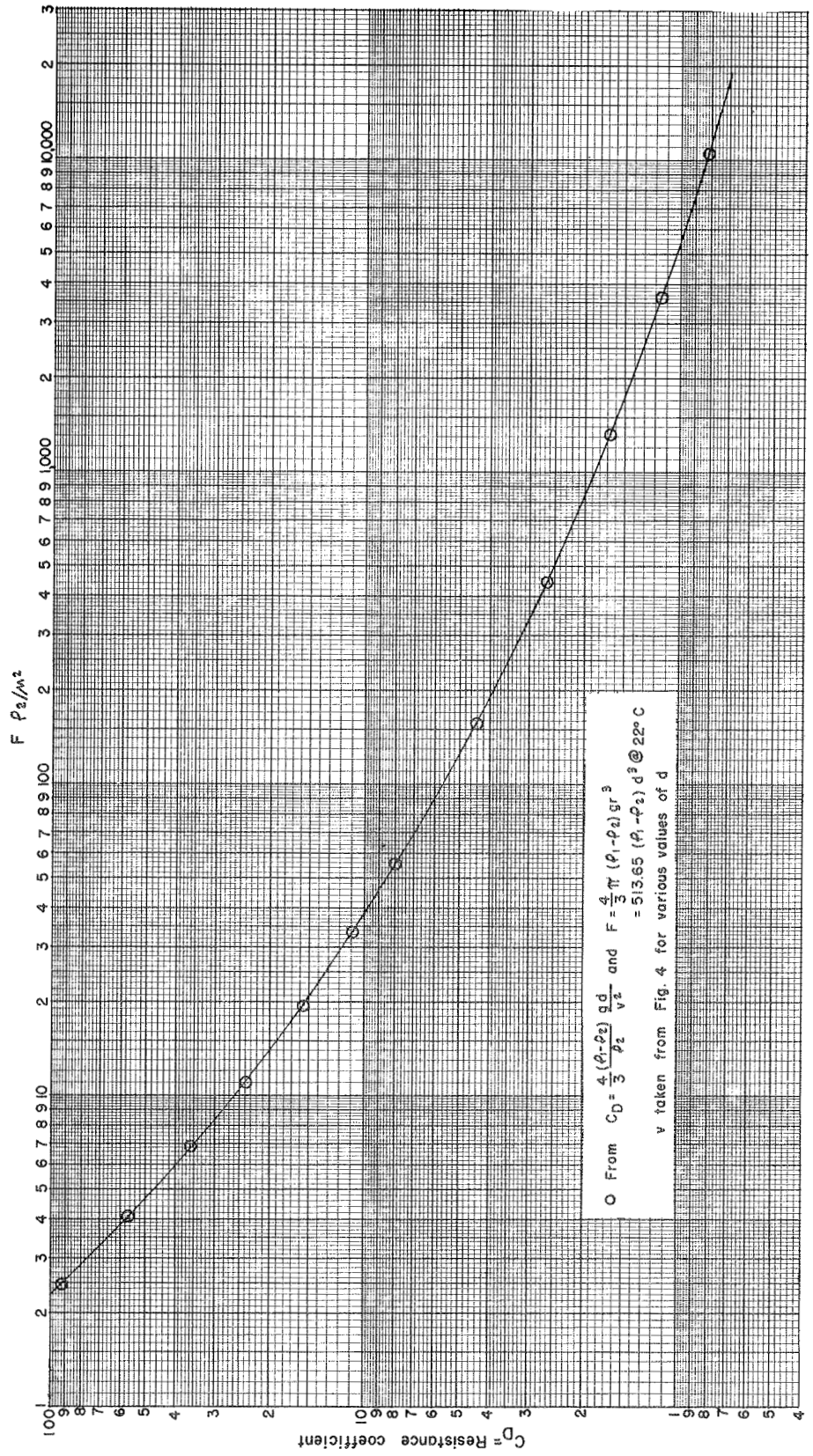


Fig. 6--Relation of resistance coefficient and F ρ_2/μ^2 based on experimental data for glass spheres

11. Preparation of glass bead test samples--The glass beads were used in compounding samples of known size distribution for the purpose of evaluating the accuracy of size analyses. A supply of the size fractions of beads retained on each of the nest of sieves was kept on hand for use in making up test samples. The main series of test samples for this investigation were made up in accordance with Table 4. Each sample was weighed out separately. The chosen weight of the coarsest size fraction was added to a dish which was tare weighted immediately prior to this addition and the total weight was recorded. Then successive increments of the progressively finer fractions were added in the same manner.

Because the sieves did not grade the glass beads with sufficient accuracy, it was necessary to correct the data obtained from the sieve grading. For example, the microscopic analysis as presented in Fig. 2 shows that at the 149 micron size, 20.5 per cent of the sieve fraction nominally larger than 149 microns was actually smaller than that size, while 4.4 per cent of the sieve fraction nominally smaller than 149 microns was actually larger than that size. The percentage finer than 149 microns as shown by sieve grading was increased by 20.5 per cent of the next coarser sieve fraction and reduced by 4.4 per cent of the next finer sieve fraction in order to obtain the actual percentage finer than 149 microns. In the same way, corrections for the errors in the sieve results were made for each of the size divisions in all of the samples used in this investigation.

12. Accuracy of the glass bead samples--Because these glass bead samples were to be used in checking the accuracy of a sedimentation

TABLE 4

SAMPLES COMPOUNDED FOR TESTING BY
BOTTOM WITHDRAWAL TUBE METHOD

Concentration	Weight grams.	Nominal Limits of Size Range in Microns				
		20-149	20-246	20-350	20-500	20-700
		Sample Numbers				
200	0.1	1,2	13,14	25,26	37,38	49,50
1000	0.5	3,4	15,16	27,28	39,40	51-55
3000	1.5	5,6	17,18	29,30	41,42	56,57
5000	2.5	7,8	19,20	31,32	43,44	58,59
7000	3.5	9,10	21,22	33,34	45,46	60,61
10000	5.0	11,12	23,24	35,36	47,48	62,63
Diameter-Microns*		Average Size Grading in Cumulative % Finer (Corrected for sieve inaccuracies by use of Fig. 2)				
691						100.0
497					100.0	90.3
345				100.0	90.3	79.3
245			99.9	90.1	77.7	64.7
171			89.8	74.6	58.1	46.8
149		99.6	82.9	66.3	50.6	39.6
124		91.2	71.3	53.8	39.8	31.2
90		56.2	36.8	27.1	19.9	16.0
77.4		37.3	23.4	19.3	13.6	11.1
64.7		21.5	14.3	11.4	8.6	8.1
45.6		2.8	2.3	2.4	2.4	2.6

Concentration is shown in p.p.m. on the basis of a volume of 500 cc.

*Values corresponding to times to fall used in the bottom withdrawal tube tests.

method for determining the size distribution within a sample, the accuracy of the sample depended on the relation of size to fall velocity.

The glass beads composing these samples were so nearly spherical in shape that no error was anticipated because of shape factors. The fractional weights of material in the samples were obtained to 0.0001 gram and, because of the weighing procedure used, errors in the fractional weights would not be cumulative. The accuracy of weighing should always be within 1.0 per cent even for samples of only 0.1 gram, and the results should be much better than that for the majority of the samples. Any errors involved in weighing the samples will be neglected.

The precision with which the sizes of glass beads in the samples were determined depends directly on the accuracy of the data in Fig. 2. Although the probable error of the microscopic determination of size distribution within a typical size fraction is 5-1/2 per cent, the maximum quantity of any of these fractions is only 20 per cent of the total sample. For the critical size fraction, 90 to 124 microns, a greater number of particles were counted, and the class limits were smaller so that the probable error was reduced to about 3-1/2 per cent for this fraction. The maximum quantity of this fraction was 35 per cent of the total sample. In either case, the indicated error would be 1 per cent of the total sample for any of the basic sample distributions of Table 4. The consistency of the data as plotted at the top of Fig. 2 also indicates that when any normal size distribution is considered, the size gradation may be determined within 1 per cent on the basis of the total weight of the sample.

The error in the calibration of the microscope is possibly as much

as 1 per cent of the particle diameter. Such an error in the diameter would indicate a similar or somewhat smaller error in size distribution in terms of the total sample weight. A total error of 2 per cent of the total weight might be involved in the size determination for these test samples.

The size distribution in terms of fall velocity also depends upon the accuracy of the relation of size and fall velocity as presented in Fig. 4 and Table 3. The density of 2.80 used to compute the fall velocity of the smaller particles actually could not have been more than 2.85 or less than 2.75, which in terms of velocity of fall indicates a range of plus or minus 3 per cent. From Fig. 4 the error in the determination of the average fall velocity for a given size of glass beads would seem to be within the same range of plus or minus 3 per cent of the fall velocity shown by the curve.

The effect of a 3 per cent error in fall velocities may be evaluated at 22° C. as follows: for a size of 149 microns the fall velocity at 22° is shown as 1.808 cm. per sec. Assuming a $\pm$ 3 per cent error, the limiting velocities would be 1.754 and 1.862 cm. per sec. corresponding to sizes of 146.3 and 151.7 microns. The errors involved in the use of these sizes instead of 149 microns may be found from Fig. 2. For 151.7 microns the percentage finer applicable to 149 microns would be in error by -15 per cent of the sieve fraction nominally contained between 149 and 175 microns and also by -2 per cent of the sieve fraction nominally between 125 and 149 microns. For the size distribution for the range from 20 to 700 microns this would mean an error in percentage finer of -15 per cent of 9.0 per cent combined with -2 per cent

of 8.2 per cent, or -1.5 per cent. The percentage finer value for 149 microns would be 1.5 per cent too low for the 151.7 micron size and the 20-700 micron size distribution. For the size distribution for the 20-350 micron samples the percentage finer for 149 microns would be 1.8 per cent too low. Similarly for the 146.3 micron size, the percentage finer values for the 149 micron size would be 1.2 and 1.5 per cent too high for the size distributions for the 20-700 and 20-350 micron samples, respectively.

The errors in per cent finer values caused by 3 per cent errors in fall velocity, evaluated on the basis of the size distributions of Table 4, are presented in Table 5. When the possible 2 per cent errors arising in the size determinations are combined with the errors shown in Table 5, a maximum error of 5 per cent is indicated for one size distribution and one critical size. Generally, maximum errors of 4 per cent would cover the samples involved. The average or probable errors would be much less than this amount. These are the possible errors in the relation of size distribution to fall velocity, the errors being expressed as a percentage of the total sample weight.

The distribution of sizes within the glass bead samples is not a normal one, but rather the concentration of beads of some sizes is relatively high while there are comparatively few beads of other sizes. While this involves no direct corrections, the lack of uniformity of size distribution may introduce some minor influences that are not sufficiently obvious to evaluate.

TABLE 5

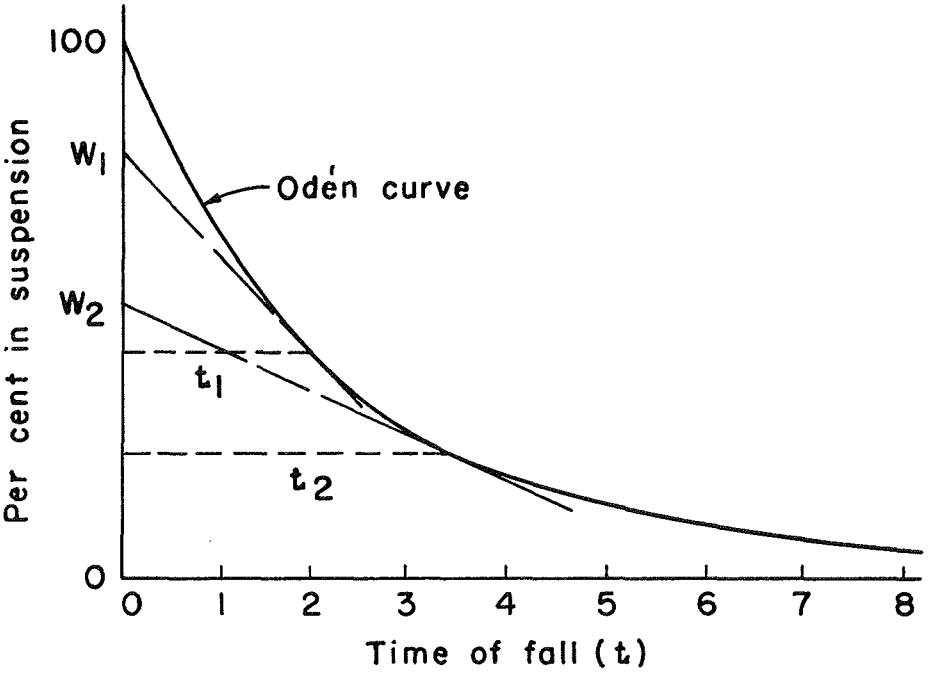
ERROR IN PER CENT FINER VALUES WITH RESPECT TO VARIATION IN FALL VELOCITY

Dividing Size Microns	Fall Velocity In Error	Difference in Per Cent Finer Figures				
		Nominal Size Range in Microns				
		20-149	20-246	20-350	20-500	20-700
497	+3%					-0.43
	-3%					+0.40
345	+3%				-0.31	-0.35
	-3%				+0.03	+0.03
245	+3%			-0.36	-0.46	-0.54
	-3%			+0.49	+0.62	+0.66
171	+3%			-0.10	-0.11	-0.11
	-3%			+0.12	+0.11	+0.11
149	+3%		-1.49	-1.76	-1.60	-1.50
	-3%		+1.27	+1.48	+1.32	+1.18
124	+3%	-0.61	-0.73	-0.77	-0.67	-0.49
	-3%	+0.83	+0.95	+0.95	+0.79	+0.59
90	+3%	-2.83	-2.10	-1.22	-1.00	-0.78
	-3%	+3.14	+2.28	+1.29	+1.07	+0.84
77.4	+3%	-1.35	-0.79	-0.67	-0.46	-0.27
	-3%	+1.65	+0.95	+0.83	+0.54	+0.31
64.7	+3%	-1.46	-0.91	-0.71	-0.46	-0.37
	-3%	+1.92	+1.20	+0.92	+0.61	+0.51
45.6	+3%	-0.06	-0.05	-0.05	-0.05	-0.06
	-3%	+0.06	+0.05	+0.05	+0.05	+0.06

III. USE OF GLASS SPHERES OF SAND SIZES TO CHECK THE ACCURACY OF BOTTOM WITHDRAWAL TUBE SIZE ANALYSES

13. Review of the bottom withdrawal tube method of size analysis--The bottom withdrawal tube method of particle size analysis is based on the theory of a uniformly dispersed sedimentation system. In 1915, Odén [5] presented his theory which was the first sound analytical approach to the reduction of dispersed sedimentation data to size gradation. The theory assumed four conditions: namely, (1) that the radii of the particles vary by infinitesimal amounts, (2) that the temperature of the system remain constant, (3) that complete dispersion of the particles be obtained, and (4) that the particles do not interfere with each other during descent. After the sediment has been dispersed throughout the sedimentation column and the settling of particles allowed to start, the accumulation at the bottom of the column at any time t will consist not only of particles with fall velocities great enough to fall the entire length of the column, but will also consist of smaller particles which had a shorter distance to fall. An accumulation curve can be plotted with time as the abscissa and percentage by weight of the total material settled out as the ordinate (the Odén curve). Whether the ordinate is expressed as percentage settled out or as percentage left in suspension is of course immaterial, and the same type of curve will result if the ordinate is in terms of weight of material. If tangents are drawn to the curve at any two points corresponding to times t_1 and t_2, and the tangents allowed to intersect the ordinate axis at W_1 and W_2, then the difference between the percentage W_2 and W_1 will represent the amount of material in a size

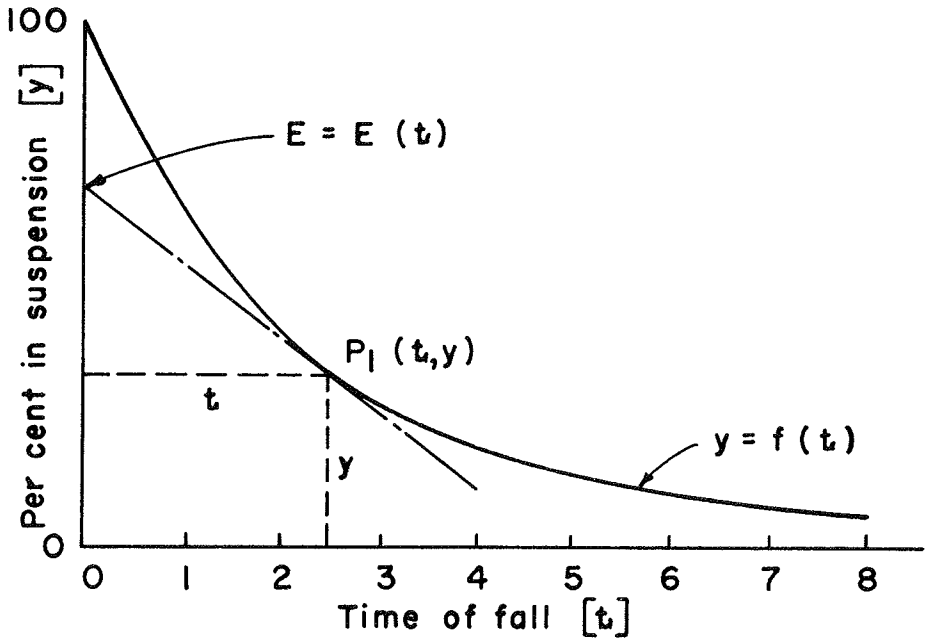
range with limits determined by the settling time t_1 and t_2.



A detailed description of the apparatus and procedure of the bottom withdrawal tube method is included in Report No. 7 of this series. In general, the tube is transparent, 48 inches in length, one inch in internal diameter, open at one end and contracted to 1/4 inch at the other. A short piece of rubber tube is slipped snugly over the small or lower end of the tube, and closed with a pinch clamp. The procedure of operation includes transferring the sediment sample to the bottom withdrawal tube, filling the tube with water to the 100 cm. level, and then inverting the tube end for end several times, allowing the air bubble to travel the length of the tube, thereby dispersing the sediment throughout the system. The tube is then placed in an upright position, timing is immediately begun and withdrawals are made from the lower end, withdrawing a known volume of suspension at predetermined time

intervals. The net weight of particles in each withdrawal is determined and an Odén curve plotted. The cumulative size frequency distribution is graphically determined by the intercepts on the ordinate axis of tangents to the Odén curve.

14. Characteristics of the Odén curve--The Odén curve based on a given size distribution can be obtained by a combination of graphical and analytical methods as will be shown in this and the following section.



The Odén curve forms the envelop for the family of tangents that cut the "y" axis at the cumulative per cent finer (E), which represents the portion of material finer than the particle size which would just settle the length of the sedimentation column in time t. The form for the slope of the curve at any point is then the slope of a general tangent and the slope may be integrated to yield the desired function $y = f(t)$.

For the tangent: $$E = -\frac{dy}{dt}\,t + y \quad \ldots\ldots\ldots\ldots \quad 5$$

or $$\frac{dy}{dt} = \frac{y - E}{t}$$

clearing, $$tdy - ydt = -Edt \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 6$$

In general, E is a function of grain size and thus of the time of fall.

$$E = E(t)$$

so that the differential equation for the Odén curve is

$$tdy - ydt = -E(t)dt \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 7$$

If $1/t^2$ be introduced on both sides of the differential equation, the left side becomes the derivative of (y/t).

$$\frac{tdy - ydt}{t^2} = -\frac{E(t)}{t^2}\,dt \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 8$$

or $$d\left(\frac{y}{t}\right) = -\frac{E(t)}{t^2}\,dt$$

integrating $$\frac{y}{t} = -\int \frac{E(t)}{t^2}\,dt + C' \quad \ldots\ldots\ldots \quad 9$$

This equation is the general functional relationship between $\underline{y}$ and $\underline{t}$ and it remains to evaluate the right hand side of the expression. For this bottom withdrawal tube investigation, samples of glass beads were synthesized from sieve fractions and the cumulative per cent finer for each sample was carefully computed. If the per cent finer (E) be plotted against the corresponding time of fall rather than against the physical diameter, the function E(t) may be determined and the integral evaluated. At once it becomes apparent that these test

samples yield an E(t) curve which breaks sharply at t_o, the time of fall corresponding to the largest grain size present in the mixture. E(t) must equal 100 per cent from zero time to this point. The Odén curve then becomes:

$$\frac{y}{t}_{t<t_o} = -100\int\frac{dt}{t^2} + C'$$

integrating $$\frac{y}{t} = \frac{100}{t} + C'$$

or $$y = C't + 100\% \quad \ldots\ldots\ldots\ldots 10$$

showing that the Odén curve is a straight line from zero time out to the time of fall of the largest grain present, and the line has a slope C'.

It is interesting to note here that the function is a valid one for a material which lacks a particular size group. If E(t) remains constant from t_1 to t_2,

$$\frac{y}{t}_{t_1<t<t_2} = -E\int\frac{dt}{t^2} + C'$$

$$\frac{y}{t} = \frac{E}{t} + C'$$

$$y = C't + E\% \quad \ldots\ldots\ldots\ldots 11$$

and the Odén curve becomes a straight line of slope C' from t_1 to t_2.

For any value of t there is a corresponding particle size, and whether this size is included in any given sample is immaterial to the accuracy of the equation as the relationships between t, dy/dt, and

E remain valid. For a size larger than the largest particle in a given sample, E is 100 per cent, dy/dt is a constant, and y varies directly with t. Whatever sizes are included in a given sample the above equation is found to apply. It should be remembered that in all discussions of this and similar methods of determining size from fall velocity determinations, the concept of size is dependent upon a continuous functional relation between actual size and fall velocity.

Although the Odén theory assumes that the radii of the particles vary by infinitesimal amounts, this restriction is not necessary to the computation of a true curve for the rate of settling of sediment having a known size distribution. This may be seen from the preceding discussion and from that on samples of discrete sizes which is yet to follow. However, the Odén curve and tangent method is usually used for the determination of size distribution from the laboratory analysis of a sample for which the distribution is not known. For this use there are practical difficulties involved in the accurate construction of the Odén curve and tangents thereto, unless the requirement that the radii of the particles vary by infinitesimal amounts is approximately satisfied.

15. Methods of computing a basic Odén curve--The first approach to the problem of computing an Odén curve for a sample of known size distribution will be made on the basis of a study of samples composed of discrete sizes. For this type of sample a rigid mathematical computation of the Odén curve is available. (See also "A Study of New Methods for Size Analysis of Suspended Sediment Samples" which is Report No. 7 of this numbered series.) As settling begins, any given size is assumed

to be distributed uniformly throughout the column. This size will then settle at a uniform rate during the time t necessary for one of its particles to fall from the water surface to the bottom. Then the rate of accumulation r for this size may be represented by

$$r = \frac{W}{t} = \frac{W}{100/v}$$

where

W = weight of size fraction,

100 = assumed fall distance in cm.,

v = fall velocity in cm./sec.

The rate of accumulation R of all sizes settling at time t is the sum of the individual rates of accumulation of all sizes equal to and smaller than that which falls a full column height in time t.

So that at time t_N the quantity settled, which is the ordinate of the Odén curve, will be:

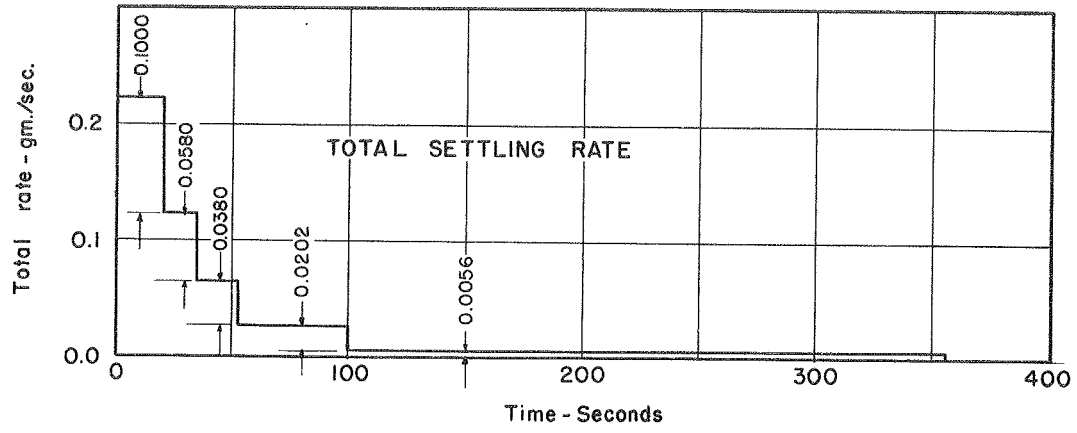
$$y_N = \sum_{n=1}^{N} R_n(t_n - t_{n-1}) \quad \ldots\ldots\ldots 12$$

Where y_N denotes the ordinate corresponding to the time of fall for size N.

A numerical example of this type of analysis for a system of discrete sizes is presented in Fig. 7. The same type of analysis based on discrete sizes may be extended to cover the problem of the computation of the Odén curve for a sample of continuous size distribution such as shown in Fig. 8. To do so, the continuous distribution is broken up into a large number of size classes. As the number of size classes is

SAMPLE COMPUTATION OF AN ODÉN CURVE FOR MATERIAL COMPOSED OF DISCRETE SIZES

1	2	3	4	5	6	7	8	9
Particle Size	Weight of each size	Fall Velocity	Fall Time 100 cm.	$t_n - t_{n-1}$	r_n Rate of accumulation	R_n Total rate	$R(t_n - t_{n-1})$	Odén Curve ordinates
microns	grams	cm./sec.	secs.	secs.	gm./sec.	gm./sec.	grams	grams
300	2	5.00	20.00	20.00	0.1000	0.2218	4.436	4.436
200	2	2.90	34.48	14.48	0.0580	0.1218	1.764	6.200
150	2	1.90	52.63	18.15	0.0380	0.0638	1.158	7.358
100	2	1.01	99.01	46.38	0.0202	0.0258	1.197	8.555
50	2	0.28	357.14	258.13	0.0056	0.0056	1.445	10.000
					Col 2/Col 4	Cum Col 6	Col 5xCol 7	Cum. Col 8



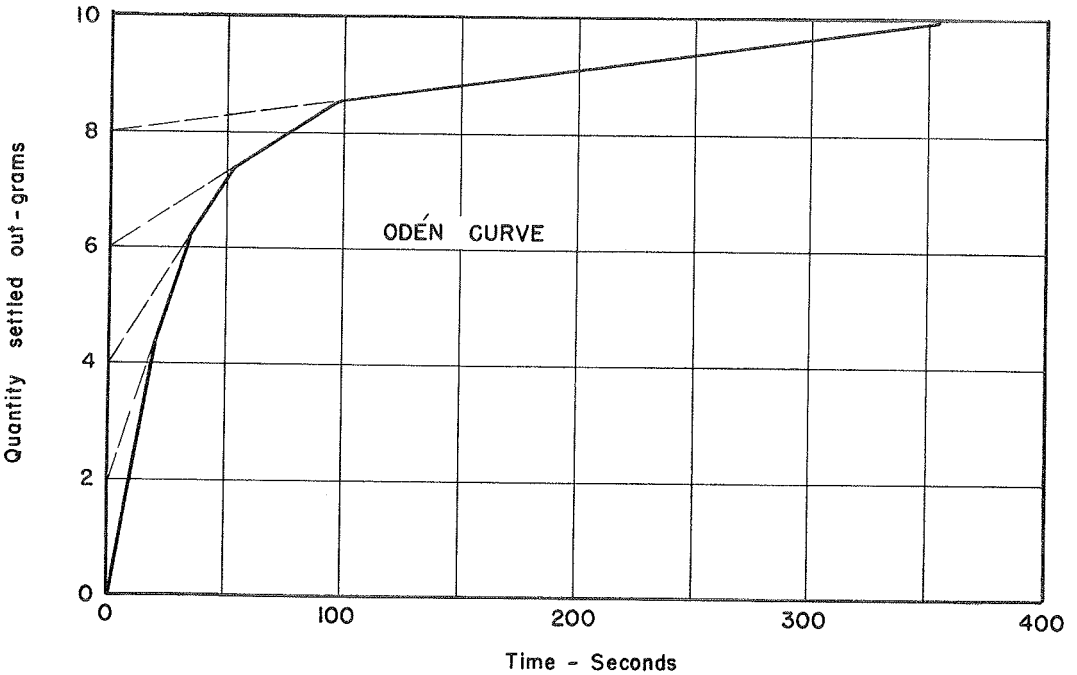


Fig. 7--Computation of an Odén curve for material composed of discrete sizes

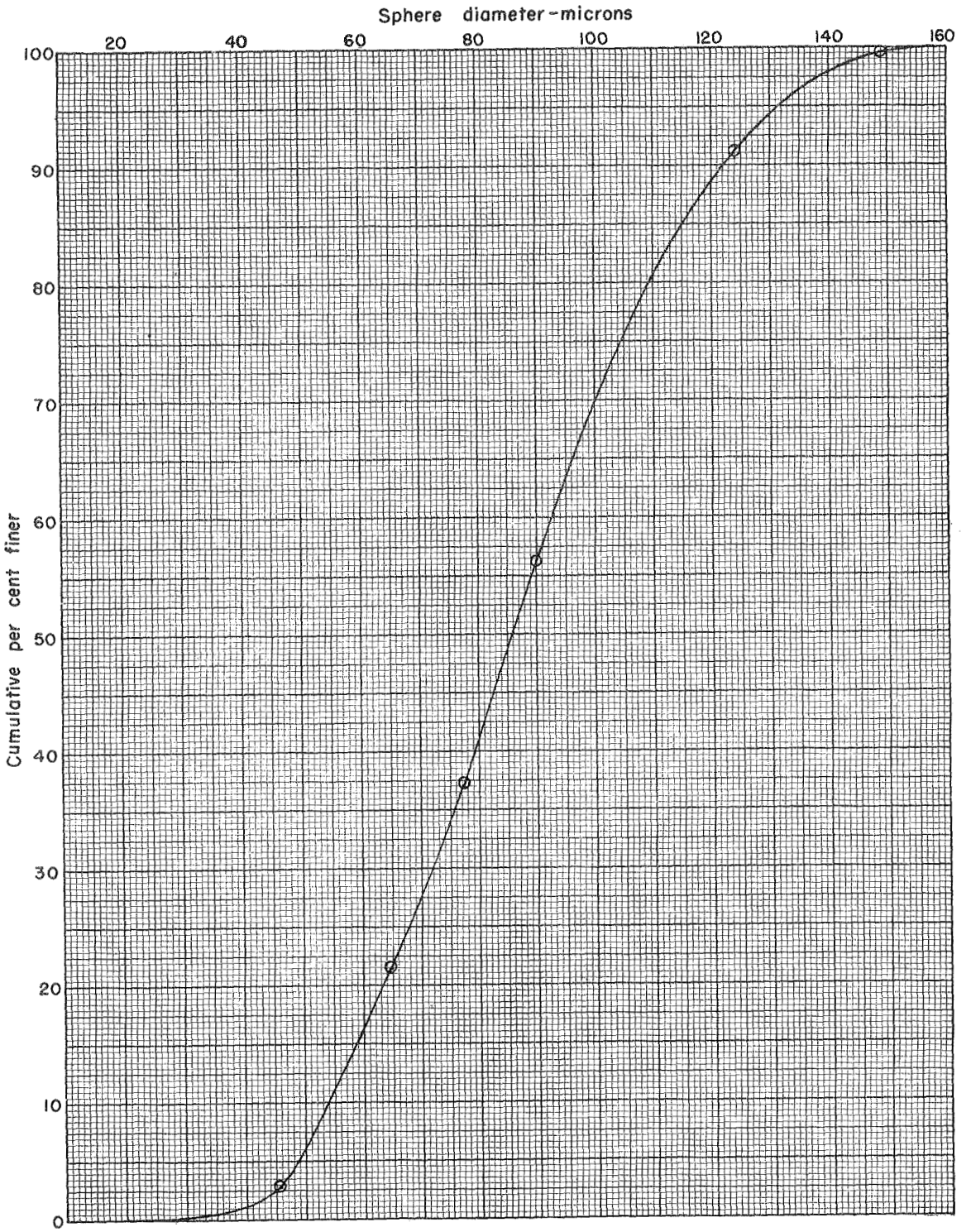


Fig. 8--Cumulative size distribution curve
20-149 microns
(Data from Table 4)

increased, the results of the approximation become more exact. To transfer the size classes into terms of discrete sizes, the graded material in each size class is considered to be replaced by an equal weight of a discrete size of particles having the same fall velocity as the average fall velocity of the graded material. The Odén curve for the sample composed of discrete sizes may be found from equation 9.

Division of a continuously graded sample into twenty size classes of equal weight has been found to give an Odén curve in fair agreement with the correct curve. However, this method of approximation assumes a condition incompatible with the actual phenomenon of settlement of a graded material. The time allotted for a fraction to settle completely is measured by the fall of the particle of average fall velocity. By the very definition of average, it is certain that there is a significant weight of particles in this class which has not reached the bottom.

The method used for computing the basic or theoretical Odén curve for the glass bead samples of this report is based on a direct analysis of the curvilinear relationships of size distribution and fall velocity, and is equally applicable to any sample for which a percentage finer curve can be established together with the fall velocities of the various sizes of sediment covered in the size distribution curve. The step by step procedure is as follows:

1. First assume a sedimentation column 100 cm. high, in which the sample will be completely dispersed prior to the start of settling. The temperature is to be considered constant throughout the period of settling, and the diameter of the tube is to be such that the height of accumulation of sediment at the bottom of the tube may be neglected. The diameter of the tube must be great enough so that the effect of the walls on the settling rate of the particles is negligible. The concentration of sediment must be low enough so that the particles fall without mutual interference.

Twenty points (a greater or smaller number could be used if desired) on the Odén curve will be determined, and the times will be the minimum times for the complete settling of the coarsest 5 per cent, 10 per cent, 15 per cent, etc., of the sample. Let these points be designated 1, 2, 3, ...20, respectively, and be represented by $\underline{N}$ which may designate any one of the numerals depending on the solution sought.

2. Plot a per cent finer curve, such as that of Fig. 8, for the sample and determine the twenty sizes s_1, s_2, s_3, $\cdots s_{20}$, such that 95 per cent of the sample will be finer than s_1, 90 per cent finer than s_2, 85 per cent finer than s_3, etc., until none is finer than s_{20}. s_{20} should be chosen as nearly as possible at the size of the finest particle, although considerable error at this point will not be serious.

3. Plot for the desired temperature, the relation between size and fall velocity for the material of the sample. For the glass beads, Fig. 9, this may be plotted as a curve based on data from Table 3. Find the velocity of fall v_1, v_2, v_3, $\cdots v_{20}$, corresponding to s_1, s_2, s_3, $\cdots s_{20}$.

4. The time t_1, in seconds for the coarsest 5 per cent of the sample to settle out may be found by dividing the maximum fall distance by v_1,

or $$t_1 = \frac{100}{v_1}$$ if velocity is in cm./sec.

also $$t_2 = \frac{100}{v_2}$$

and $$t_N = \frac{100}{v_N}$$

which is the general equation for the time in seconds at which all the sediment having a fall velocity greater than v_N will have settled to the bottom of the tube,

also $$T_N = \frac{100}{60 v_N} = \frac{5}{3 v_N}$$

when T_N equals the time in minutes.

5. Therefore, at time t_N the total amount of material having a fall velocity greater than v_N that will have completely settled will be

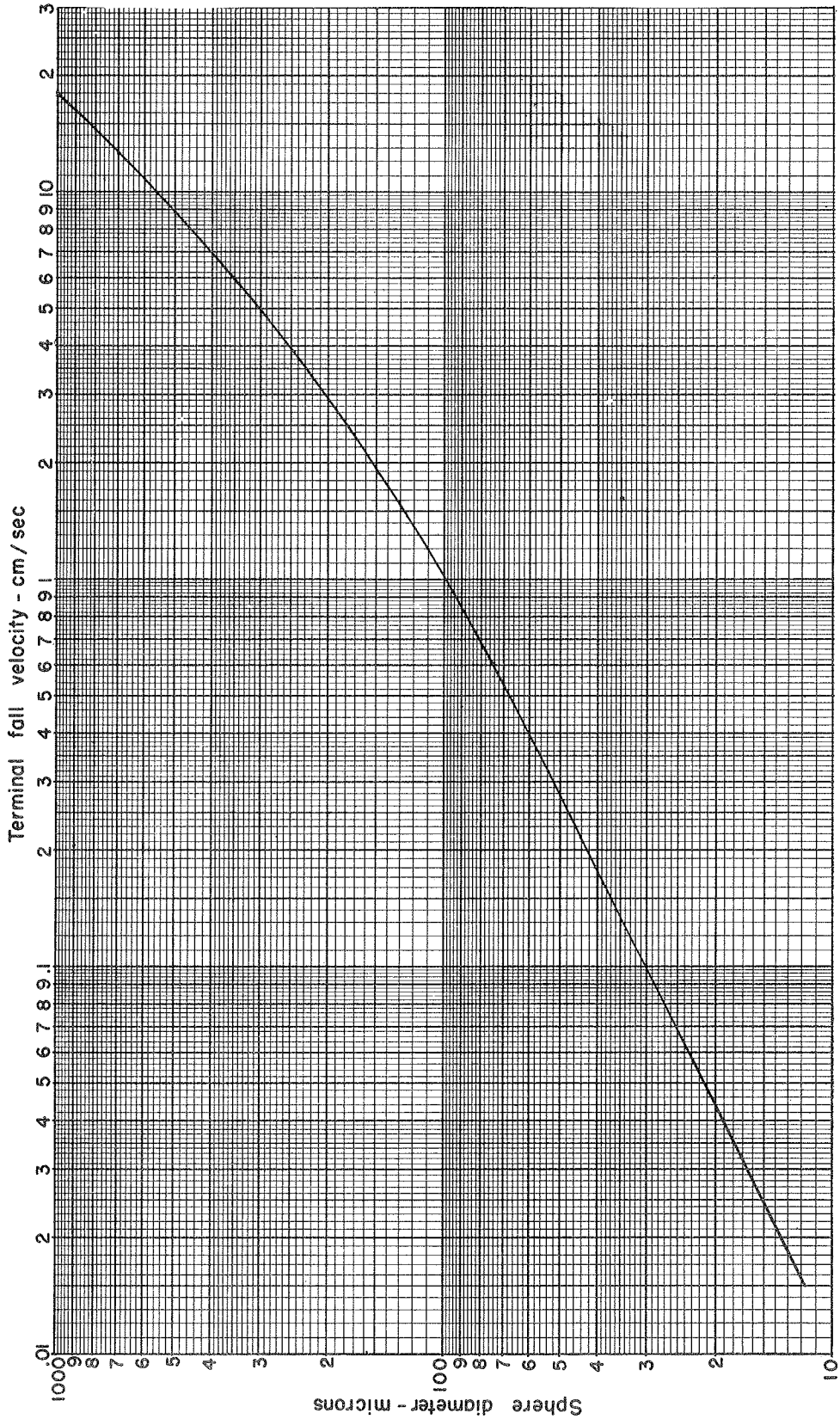


Fig. 9--Terminal fall velocity of glass spheres in water at 25° C. Computed from data in Table 3

5% when N = 1
10% when N = 2
or in general 5N%.

If units of per cent are considered equivalent to units of weight the coarser fraction completely settled in time t_N equals 5N.

6. In addition to the coarser material completely withdrawn, some of the finer fractions (having a fall velocity less than v_N) will have settled to the bottom of the tube.

At any time t_N a given size of particle will have fallen a distance equal to t_N times its fall velocity. Then that is the distance at the top of the sedimentation column which will have been completely cleared of the given size of sediment. The cleared distance divided by the original height of column, or 100 cm., will be the fraction of the original material of that size that will have settled to the bottom of the sedimentation tube. Allowing $\frac{v_N + v_{N+1}}{2}$ to equal the average fall velocity of the fraction next finer than s_N, the weight of that fraction settled out will be

$$\frac{t_N}{100}\left[\frac{v_N + v_{N+1}}{2}\right] 5$$

For all fractions finer than s_N the total weight settled out will be

$$\frac{t_N}{100}\left[\frac{v_N + v_{N+1}}{2} + \frac{v_{N+1} + v_{N+2}}{2} \cdots + \frac{v_{19} + v_{20}}{2}\right] 5$$

or $$\frac{t_N}{40}\left[v_N + 2(v_{N+1} + v_{N+2} \cdots + v_{19}) + v_{20}\right]$$

or $$\frac{t_N}{40}\left[2(v_N + v_{N+1} + v_{N+2} \cdots + v_{20}) - v_{20} - v_N\right]$$

or $1.5\, T_N\, Q$ when T_N equals time in minutes

and $$Q = 2\sum_{n=N}^{n=20} v_n - v_{20} - v_N$$

7. The total weight of sediment settled out by the time T_N is, therefore, the sum of the two general expressions from steps 5 and 6:

$$5N + 1.5\ T_N Q.$$

If this value is subtracted from 100, it will give the weight of sediment remaining in suspension or

$$100 - 5N - 1.5\ T_N Q.$$

8. The term $100 - 5N$ may be rapidly evaluated for the values of N from 1 to 20. As this term is independent of the size distribution and fall velocity, the same values always apply without regard to the kind of sample involved.

The term $1.5\ T_N$ may be computed for the twenty values of N from

$T_N = \frac{5}{3v_N}$ the relation developed in 4 above.

The twenty values of Q may be obtained as follows:

The value for $N = 19$ is $v_{19} + v_{20}$.
The value for $N = 18$ is $v_{18} + 2\ v_{19} + v_{20}$.

That is, having the value for N equals 19 to get that for N equals 18, add $v_{18} + v_{19}$.

or having the value of Q for N, add thereto

$v_{N-1} + v_N$ to get the value for $N - 1$.

9. Having determined the sediment remaining in suspension at times t_1 to t_{20}, it is possible to plot the Odén curve for the sample. The basic Odén curve of Fig. 10 was plotted from the computations shown in Table 6. The derivation and computations were made and the curve plotted in terms of sediment remaining in suspension because the results of the laboratory tests were presented in this way.

16. Program for testing the bottom withdrawal tube method--The experiments forming the basis for Report No. 7 of this series dealt extensively with sediments of grain sizes less than 62.5 microns. Comparison with the pipette method indicated that, for these small sizes, the bottom withdrawal tube yielded accurate and consistent results.

TABLE 6

COMPUTATIONS FOR BASIC ODEN CURVE FOR SIZE RANGE 20 TO 149 MICRONS

1	2	3	4	5	6	7	8	9	10
N	s_N microns	v_N cm/sec	v_N+v_{N+1}	Q	T_N min.	$1.5T_N$	$1.5T_NQ$	100-5N	% in 100 cm.
1	132	1.60	3.01	29.48	1.042	1.563	46.08	95.00	48.92
2	122	1.41	2.70	26.47	1.182	1.773	46.93	90.00	43.07
3	115	1.29	2.49	23.77	1.292	1.938	46.07	85.00	38.93
4	110	1.20	2.30	21.28	1.389	2.084	44.35	80.00	35.65
5	105	1.10	2.12	18.98	1.515	2.272	43.12	75.00	31.88
6	100	1.02	1.98	16.86	1.634	2.451	41.32	70.00	28.68
7	97	.96	1.85	14.88	1.736	2.604	38.75	65.00	26.25
8	93	.89	1.71	13.03	1.873	2.810	36.61	60.00	23.39
9	89	.82	1.59	11.32	2.033	3.050	34.53	55.00	20.47
10	86	.77	1.49	9.73	2.164	3.248	31.60	50.00	18.40
11	83	.72	1.38	8.24	2.315	3.472	28.61	45.00	16.39
12	79	.66	1.27	6.86	2.525	3.788	25.99	40.00	14.01
13	75	.61	1.17	5.59	2.732	4.098	22.91	35.00	12.09
14	72	.56	1.06	4.42	2.976	4.464	19.73	30.00	10.27
15	68	.50	.94	3.36	3.333	5.000	16.80	25.00	8.20
16	63	.44	.82	2.42	3.788	5.682	13.75	20.00	6.25
17	59	.38	.70	1.60	4.386	6.579	10.53	15.00	4.47
18	54	.32	.59	.90	5.208	7.812	7.03	10.00	2.97
19	49	.27	.31	.31	6.173	9.260	2.87	5.00	2.13
20	20	.04	--	--	41.667	62.500	--	0.00	0.00
	From Fig. 8	From Fig. 9	See Col. 3	Cum. of Col. 4	$\frac{5/3}{\text{Col. 3}}$	1.5 x Col. 6	Col. 5 x Col. 7	100 - 5 x Col.1	Col. 9 -Col. 8

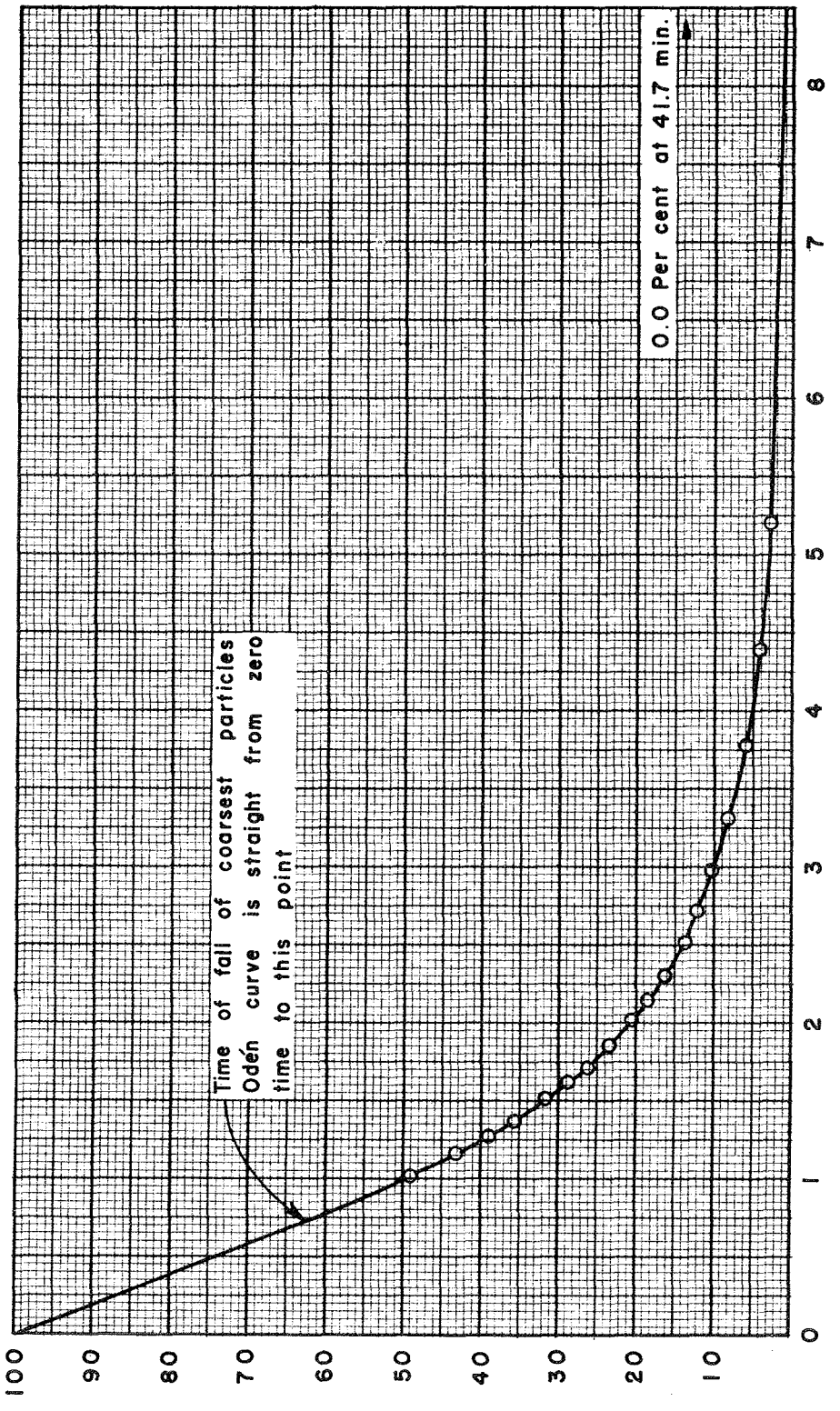


Fig. 10--Basic Odén curve for size range 20 to 149 microns

However, because of the national emergency, the testing program was curtailed when only a few tests had been made to determine the effectiveness of the method in analyzing sands. The accuracy of the method was questionable when applied to sediments in the medium and coarse sand sizes. It seemed necessary, therefore, to make detailed tests to check the accuracy of the bottom withdrawal tube method for size analysis of samples which are composed predominantly of particles in the sand sizes.

Samples characterized by five different size distributions, each having a fine limit of about 20 microns and different coarse limits varying from 125 to 700 microns, were compounded from sieved fractions of glass beads. Each size distribution was made up in six different sample weights which, on the basis of a 500 ml. suspension, correspond to concentrations of from 200 to 10,000 p.p.m. by weight. Six separate samples were compounded for each combination of size range and concentration and each sample was made up individually by weighing out the required amounts of each sieve fraction. Pairs of samples from each set of six were then analyzed at each of three different laboratories by personnel normally engaged in size analysis work, thereby extending the experimental program to cover differences in techniques and personal accuracy. The three laboratories submitted a complete analysis of each sample showing the final cumulative size distribution. Table 4 shows the size distribution and the concentration of each of the basic samples prepared for this testing program.

17. Modifications of bottom withdrawal tube procedure--The bottom withdrawal tube method, as explained in Report No. 7, was followed by each of the laboratories analyzing samples for this investigation,

except for differences in the methods of making and using the results of the final withdrawals. Theoretically, from the sizes of material used in these samples, there should have been practically no material in the final withdrawal. However, when this last withdrawal was made in the usual manner, that is, completely withdrawn, the material in the final withdrawal was consistently greater than that in the previous one. Obviously the final withdrawal contained material which had been supported on the meniscus, or which was washed from the shoulders of the tube during the last withdrawal. Any material which was drained or rinsed from the tube after the final withdrawal was not included as a part of the analysis. If material contained in the rinse or excess in the final withdrawal is equally representative of all size fractions it may be disregarded entirely without changing the size distribution found in the analysis. If the material is preponderantly of the finer sizes, the omission will lead to erroneous results. Microscopic inspection has indicated that the excess material found at the end of analyses similar to those reported here represents a wide range of sizes, but this evidence is not conclusive enough to satisfy completely the needs of this discussion. See also Section 22.

The laboratory data for the test samples were processed as follows:

In laboratory "A" the procedure of Report No. 7 was followed and the final withdrawal was used in determining the Odén curve for each sample. However, when the data from laboratory "A" were plotted as a check against the basic Odén curve, the amount of material in the last withdrawal was considered a part of the residue and as such was not included. The series of samples in the 20-700 micron size range was not

completed in laboratory "A." The series was later processed in laboratory "B" and substituted for the incomplete data.

The procedure of Report No. 7 was followed in laboratory "B," except that, based on the reasoning that material held on the meniscus did not fall in accordance with the assumptions of the Odén theory, the final withdrawal was stopped while the meniscus remained in the neck of the tube. The final withdrawal made in this way contained very little material, but what there was has been included when computing the Odén curve for each sample, and also when comparing with the basic Odén curve.

In laboratory "C," about twenty samples were processed according to the methods of Report No. 7, except that material in the last withdrawal was considered a part of the residue and was not included in determining the Odén curve for each sample nor was it used in comparing results with the basic Odén curve. The remaining samples were processed in the same manner employed by laboratory "B."

The Odén curves for the individual samples were drawn up by the personnel of the laboratory making the analyses and the methods used are merely reported here. The basic Odén curves were computed as a part of this report and are the curves defined by the composition of the glass bead samples made up for use in these analyses. The results from the laboratories have been compared with the basic Odén curves by the methods which seem to provide the most direct check on the accuracy of the laboratory work. The differences resulting from the method of treating the final withdrawal are not serious for the purpose of this report.

18. Adjusting bottom withdrawal tube data to a common temperature--Methods for computing the basic or theoretically correct Odén curve for

a sediment sample of known size distribution have been given. However, this curve varies slightly with changes in temperature. In this investigation the bottom withdrawal tube analyses were made in laboratory "A" at a temperature of about 30° C., in laboratory "B" at about 25° C., and in laboratory "C" at 26° to 29° C. In order to make the data from the laboratories directly comparable, the data from laboratories "A" and "C" were corrected to the basis of 25° C. by the following method:

1. For a given size of particle and over a limited temperature range, the time to fall 100 cm. can be assumed to be a linear function of temperature. This relationship is shown in Fig. 11, the data for which were taken from the fall velocities in Table 3.

2. In Fig. 12 the rates of change of time of fall with respect to temperature (the slopes from Fig. 11) were plotted against time to fall 100 cm. at 25° C. Two curves emerge that depend upon the size of particle. Times of fall computed by Stokes' equation yield a straight line for values of time from 3.9 to 60 min. Shorter times of fall based on the impact formula conform to an exponential relation:

$$\frac{dt}{dT^{\circ}} = \frac{t_{25}^{5/4}}{62} \quad \text{for time from 0 to 3.9 min.}$$

Where $\frac{dt}{dT^{\circ}}$ equals the change in time to fall 100 cm. in terms of temperature change in degrees Centigrade and t_{25} equals the time to fall 100 cm. at 25° C.

3. From the physical aspects of the Odén curve, the abscissa at any point is the "time to fall 100 cm." of the smallest particle in the completely sedimented fraction of the sample. So that if, for the same sample, the temperature of the settling media be decreased, this time of fall will be increased and every point of the resulting Odén curve would be translated in time by an amount indicated in paragraph 2. The above method is not entirely precise, but its applicability was checked by computing a basic Odén curve for a 30° temperature which showed the same displacement as found by the method outlined.

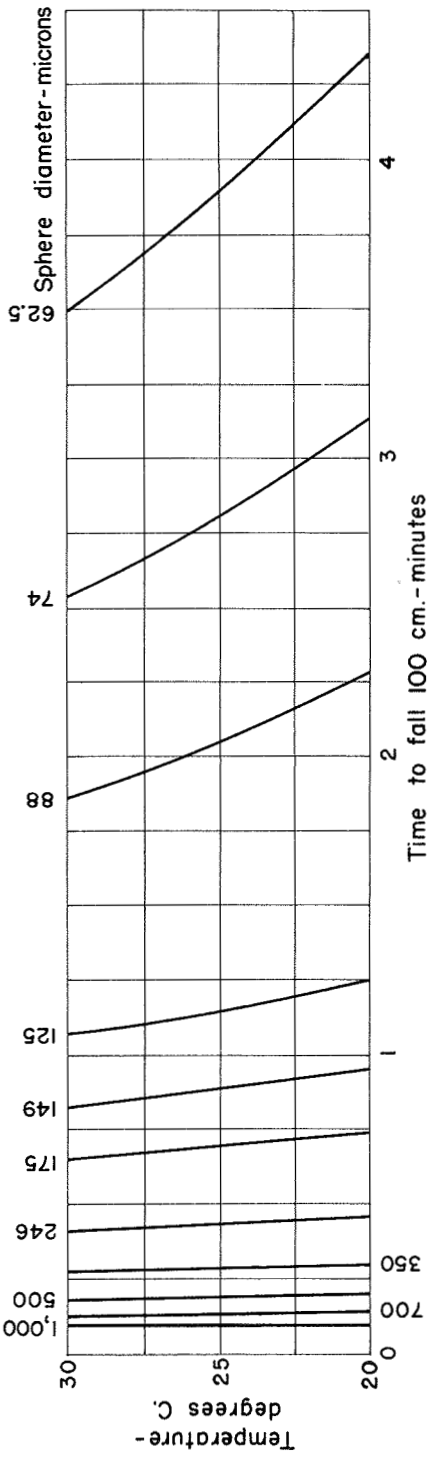


Fig. 11a--Time for glass spheres to fall 100 cm. in water under conditions of impact law

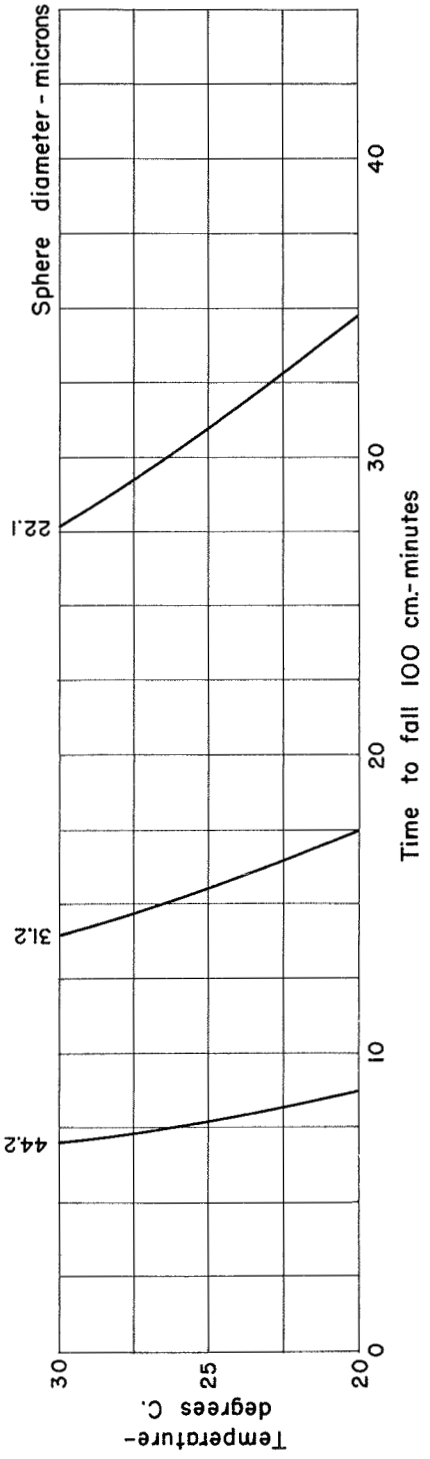
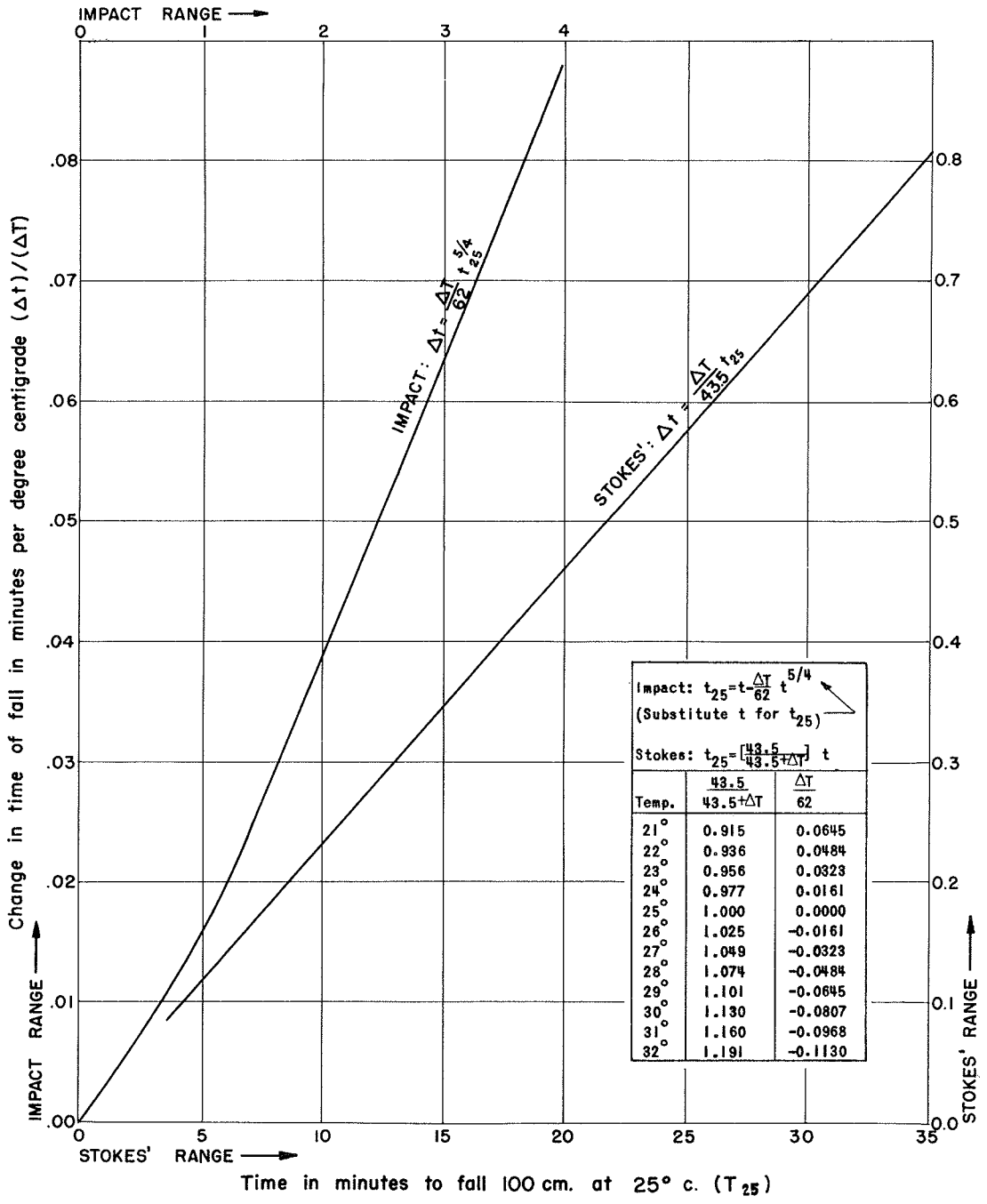


Fig. 11b--Time for glass spheres to fall 100 cm. in water under conditions of Stokes' law



Temp.	$\frac{43.5}{43.5+\Delta T}$	$\frac{\Delta T}{62}$
21°	0.915	0.0645
22°	0.936	0.0484
23°	0.956	0.0323
24°	0.977	0.0161
25°	1.000	0.0000
26°	1.025	-0.0161
27°	1.049	-0.0323
28°	1.074	-0.0484
29°	1.101	-0.0645
30°	1.130	-0.0807
31°	1.160	-0.0968
32°	1.191	-0.1130

Fig. 12--Temperature corrections applicable to time to fall 100 cm. at 25° C.

Temperature corrections were applied to the times of fall of all data before comparing with the basic Odén curves.

19. Comparison of experimental data with basic Odén curve--The Odén curves shown in Figs. 13 to 17 give a true picture of the errors involved in the basic data determined in the bottom withdrawal tube method of size analysis of glass bead samples. These errors are the ones existing prior to the usual graphical analysis for the determination of the cumulative size distribution.

A basic Odén curve for 25° C. has been computed for each size range on the basis of the size distribution in the test samples. The basic curve is flanked by two curves which give a uniform error of plus and minus 5 per cent in cumulative size distribution. Accordingly, every experimental curve having the shape of the basic curve and lying between the two outer curves must yield a cumulative size distribution within 5 per cent of that of the test sample. Obviously, 100 per cent of the sample should be in suspension at zero time, and none of the sample should be in suspension after the time for the smallest particle present to fall 100 cm.

One of the first trends apparent in the experimental data is the tendency for the first withdrawal to increase in error as the size of beads involved increases. When the maximum size of particles involved is 149 microns, the results of the first withdrawal appear quite satisfactory especially those from laboratory "A"; with a maximum size of 246 microns the results of the first withdrawal become somewhat erratic; at a maximum size of 350 microns the results are so undependable as to be almost useless; and when the maximum size of beads in the sample is 700

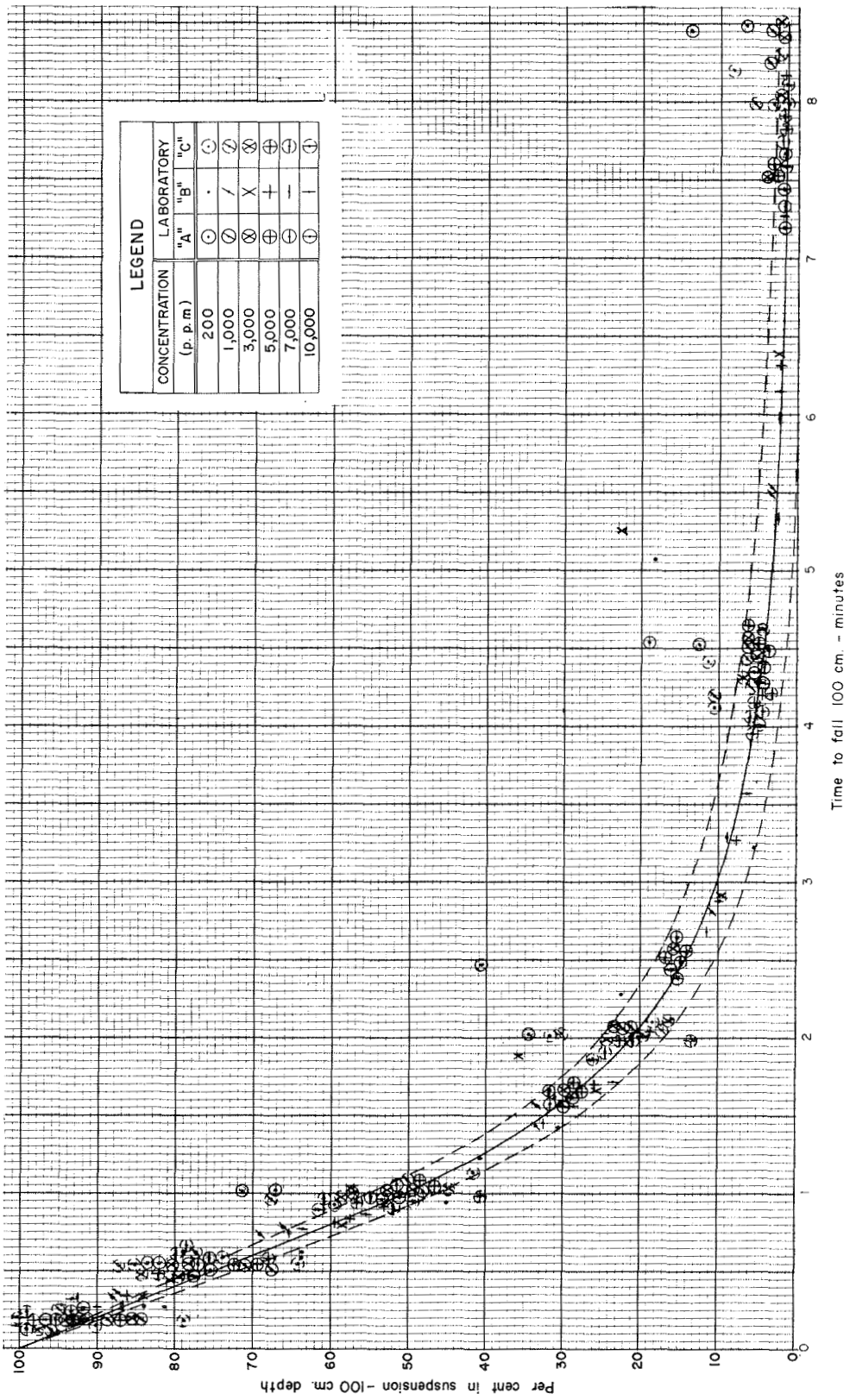


Fig. 13--Comparison of experimental data with basic Odén curve for size range 20 to 149 microns

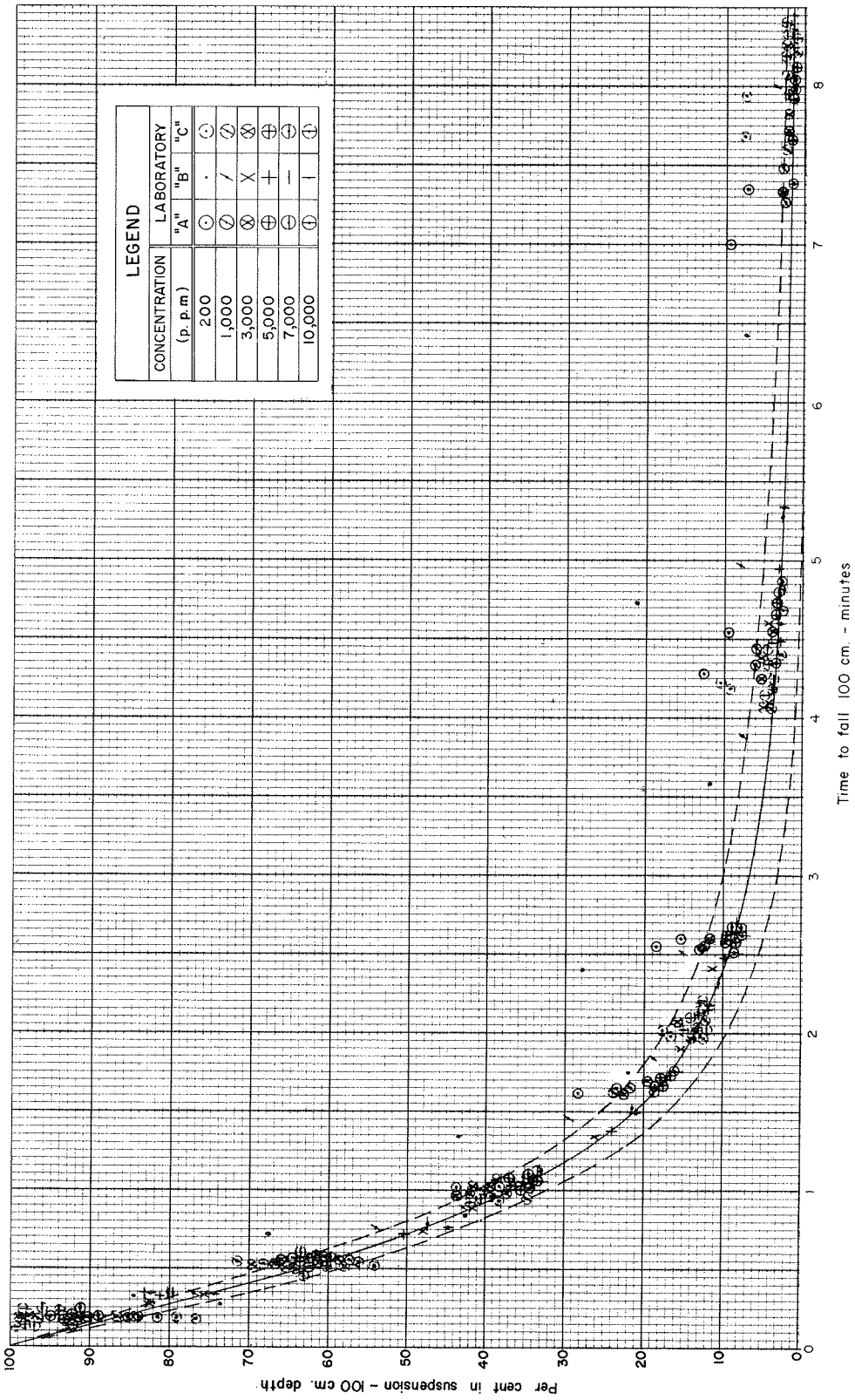


Fig. 14--Comparison of experimental data with basic Odén curve for size range 20 to 246 microns

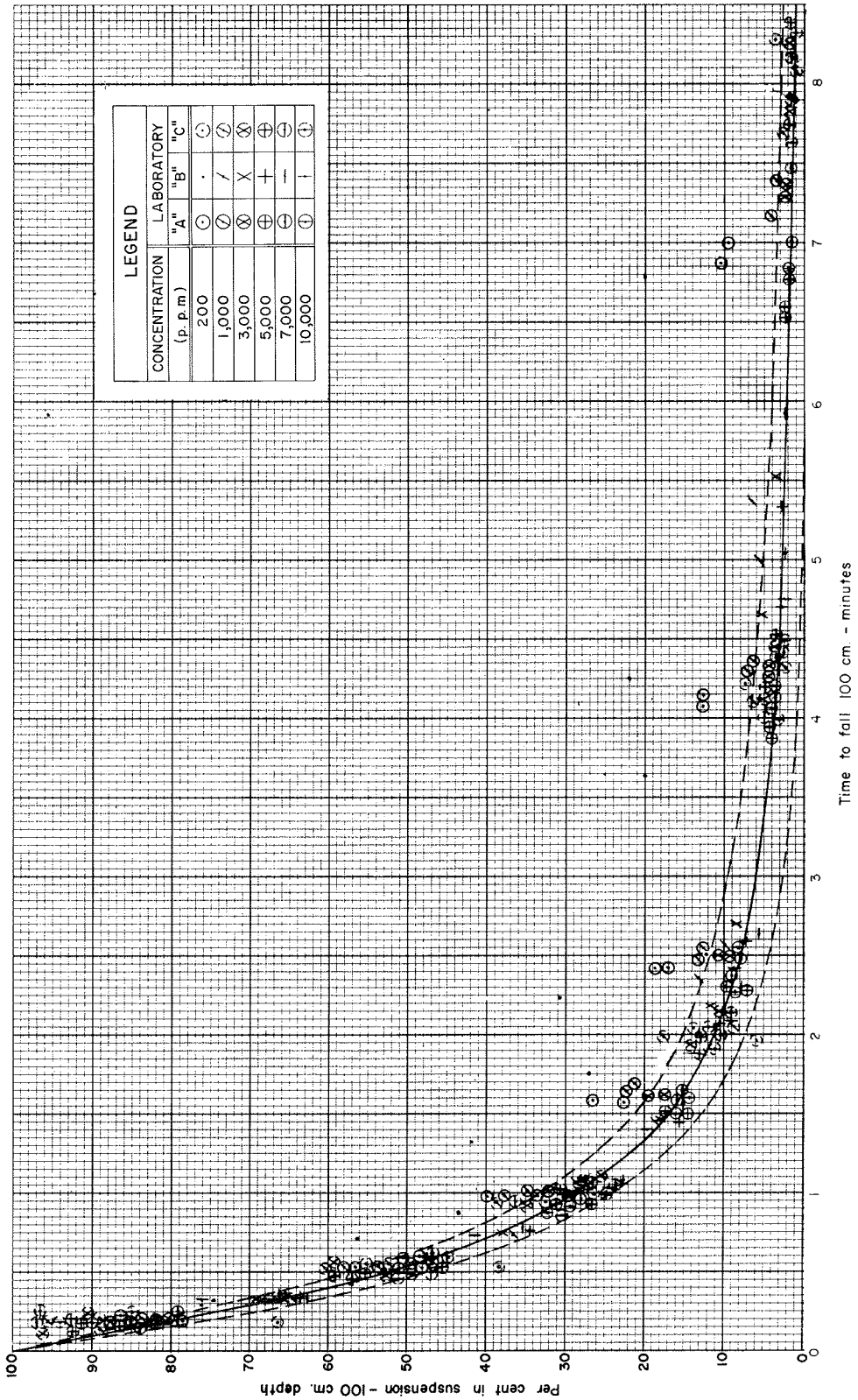


Fig. 15--Comparison of experimental data with basic Odén curve for size range 20 to 350 microns

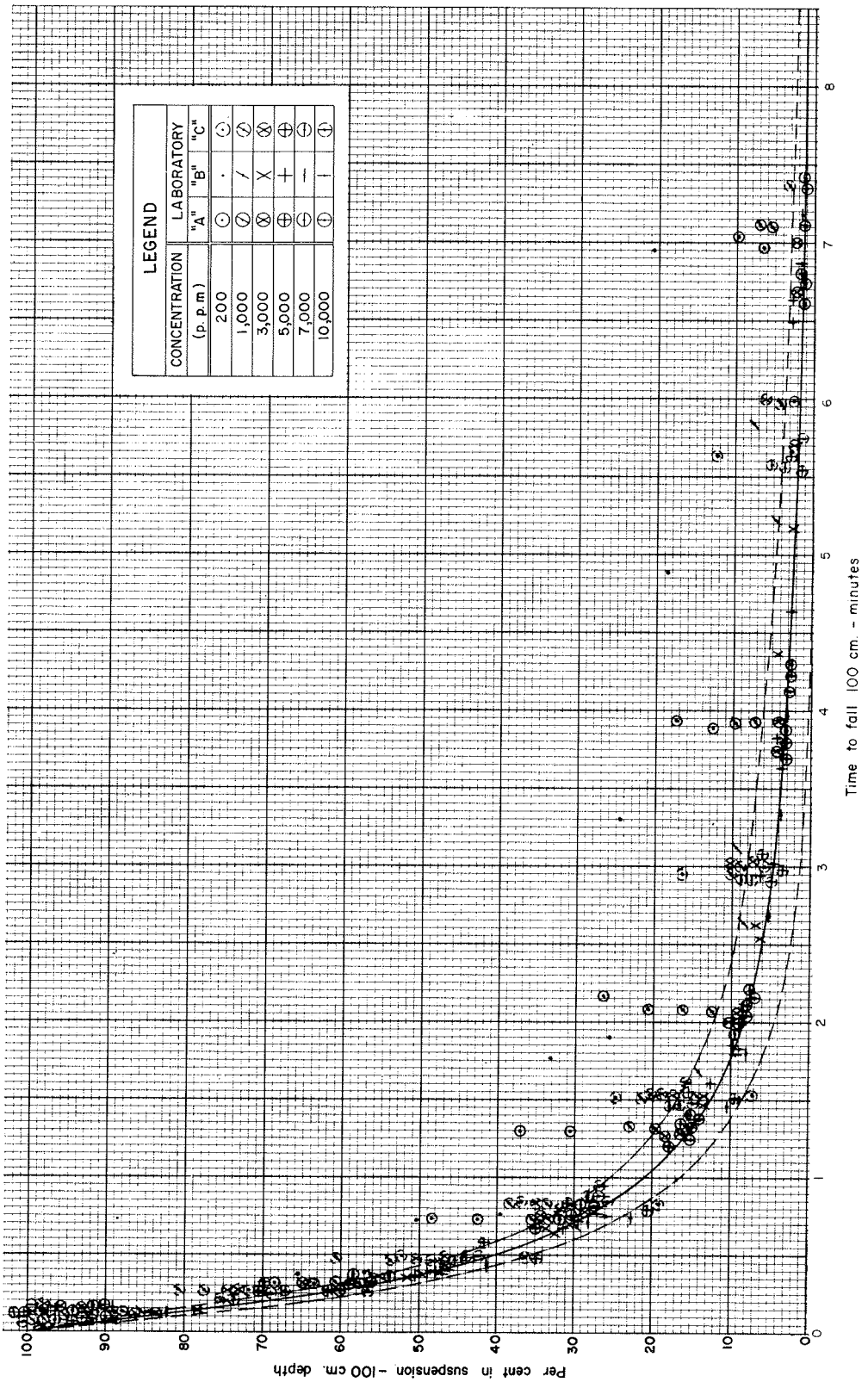


Fig. 16--Comparison of experimental data with basic Odén curve for size range 20 to 500 microns

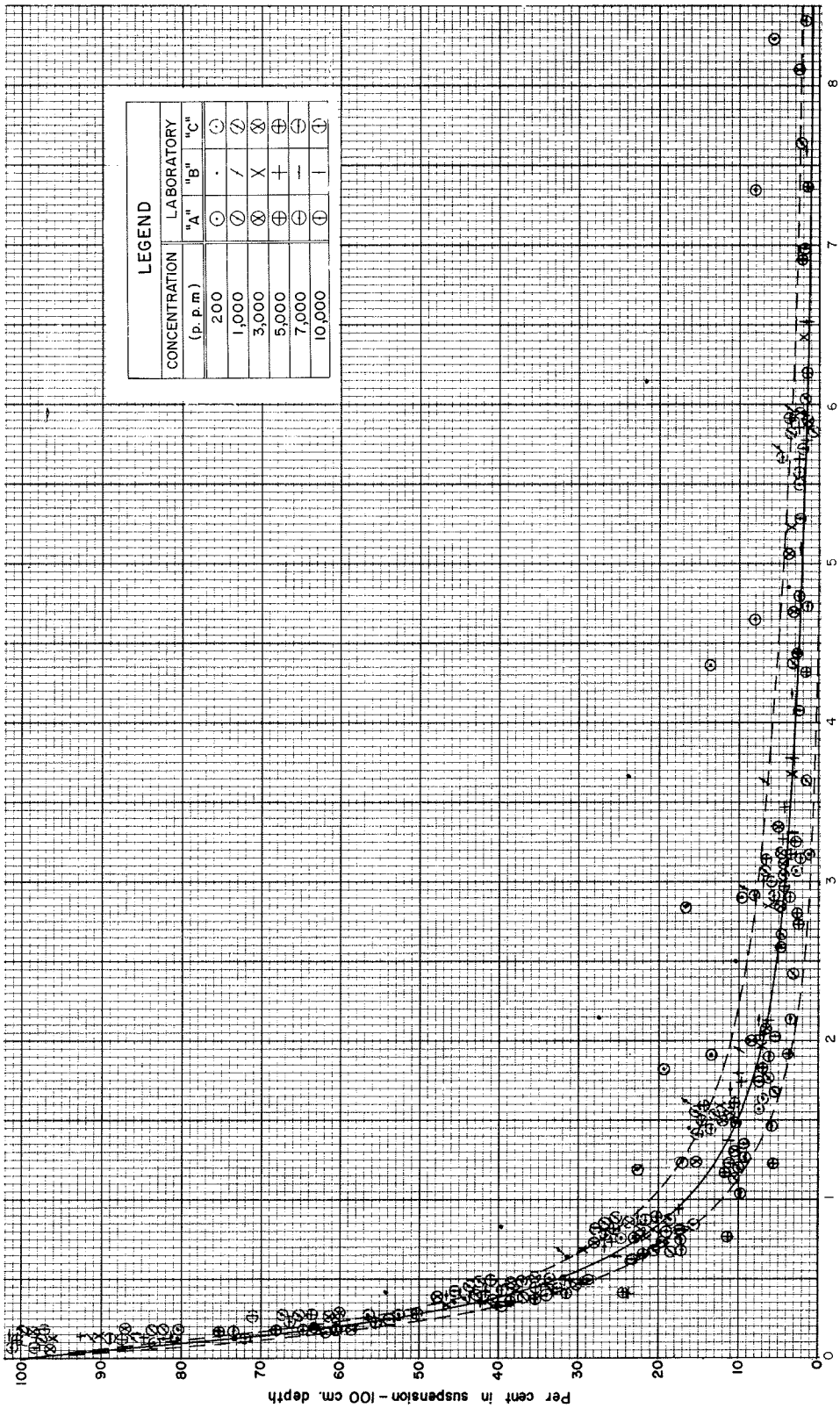


Fig. 17--Comparison of experimental data with basic Odén curve for size range 20 to 700 microns

microns, the first withdrawals are entirely unsatisfactory, especially those made in laboratories "A" and "C." As the Odén curve for sand sizes cannot contain a reversal of curvature, whenever the first withdrawal could not be included on a smooth curve, it was disregarded. The procedures in all laboratories were consistent in such cases, and the data from the second withdrawal were combined with a smooth curve drawn to 100 per cent in suspension at zero time. The important consideration seems to be that, as increasingly larger sizes of beads are contained in the sample, the present laboratory technique fails to obtain a sedimentation regime consistent with the Odén theory of sedimentation in a dispersed system. The error involved undoubtedly is carried over into the remainder of the Odén curve, but gradually becomes decreasingly important as the first withdrawal becomes a smaller portion of the total accumulation of sedimentated materials.

A second difficulty appears in the results for samples that have a concentration of 200 parts per million. A study of the data reveals that results obtained at these concentrations are not as consistent as those obtained at the higher concentrations. However, probably far more important is the fact that the data usually plot high above the basic Odén curve, an indication of repetitious error. For the results from laboratories "A" and "B" the sum of the weights of the withdrawals exceeded the weight of the original sample analyzed, and in some cases the excess was as great as 50 per cent of the original sample. The data from laboratory "C" frequently showed the sum of the withdrawals to be much less than the weight of the original sample. This indicates that the techniques employed by the laboratories are not precise enough to

give acceptable accuracy to the size analyses of samples of low concentrations. Probably the error in weight is about the same for each withdrawal and, consequently, most obvious when there is very little material present in the withdrawal. These discrepancies still appear at concentrations of 1,000 p.p.m., and even at this concentration the results obtained are of questionable accuracy. Presumably this type of error runs through all the analyses, but at the higher concentrations the percentage error becomes small enough to be unimportant, at least in those withdrawals which contain appreciable weights of material. In some of the last withdrawals, even in the samples of higher concentrations, there may be so little material that this error in the weight may be a factor that causes the data to plot wide of the basic Odén curve. The use of the depth factor in the bottom withdrawal tube method greatly magnifies errors in the final withdrawals.

In general, the comparison of the experimental data with the basic Odén curve leads to the conclusion that the results of analyses at 200 p.p.m. are unacceptable, at 1,000 p.p.m. a large majority of the results fail to fall within the 5 per cent curves. At 3,000 to 10,000 p.p.m. most of the results come within these limiting 5 per cent curves. However, the inaccuracies involved in drawing Odén curves and tangents for individual sample analyses will cause most individual analyses to show errors greater than 5 per cent. This is especially true when coarser material is present and the first withdrawal cannot be used to define the Odén curve. Usually when the coarser sand sizes of beads are present the per cent finer curve will show excessive percentages.

Within the limits of these tests the apparent accuracy of results

is not adversely affected by increasing the concentration of material in the bottom withdrawal tube. The results found at a concentration of 10,000 p.p.m. are as good as those at any other concentration. The increased weights of material handled at the higher concentrations reduce the percentage error involved in laboratory techniques.

While these tests indicate that a concentration of 10,000 p.p.m. is not too high for glass bead samples in the sand size ranges, and is perhaps not too high for sand samples, experience has shown that for sediment samples containing silts and clays the maximum concentrations used in a bottom withdrawal tube analysis should generally not exceed 3,500 p.p.m. For samples containing silts and clays, these limits are important and must be recognized in order to obtain satisfactory analyses.

20. Accuracy of bottom withdrawal tube analyses of glass beads--In addition to the types of errors involved in the laboratory data discussed in the previous section, inaccuracies in the results of the bottom withdrawal tube analyses may arise from the determination of the size gradation by means of the Odén curve and tangent method. Because this method is an integral part of the analysis, the size gradations were used just as submitted, and without correction of any kind, as the basis for evaluating the accuracy actually obtained by the bottom withdrawal tube method in three different laboratories. The only way of determining the magnitude of the errors introduced into this series of analyses by the curve and tangent method is by a comparison of the errors indicated in the previous section with those presented in this section which deals with the total errors and not just those based on laboratory technique. A general discussion of the errors inherent in the

curve and tangent method may be found in Section 23.

The best method of evaluating the accuracy, or the extent to which the results of the bottom withdrawal tube analyses satisfy the need, will depend upon the application which is to be made of the data. An analysis may give the correct median grain size and still be in error at many points, it may give the correct amounts of many of the size fractions even though the per cent finer curve is seriously in error, and the average results of several analyses may be quite accurate but the individual analyses may be erratic--there are many possibilities which could be considered.

A study of the accuracy of the analyses will first be made on the basis of the differences in the per cent finer figures for the original glass bead sample and the per cent finer figures resulting from the bottom withdrawal tube analysis of the same sample. The results of all tests are shown in Figs. 18 to 23. Each figure shows results for samples of the same concentration, with the results plotted separately for each size range. A positive deviation at a given size indicates that the bottom withdrawal tube analysis shows a greater per cent finer at that point than was given for the base sample. These figures provide a means of studying not only the numerical accuracy of the results, but also the consistency in terms of plus and minus values, locations of maximum and minimum errors with respect to size and concentration, etc.

Table 7 is presented to aid in interpreting the effect of size range and concentration on the accuracy of the results of individual analyses. Separate counts were made of all tests which showed no variation in per cent finer of more than 5 per cent from the base sample;

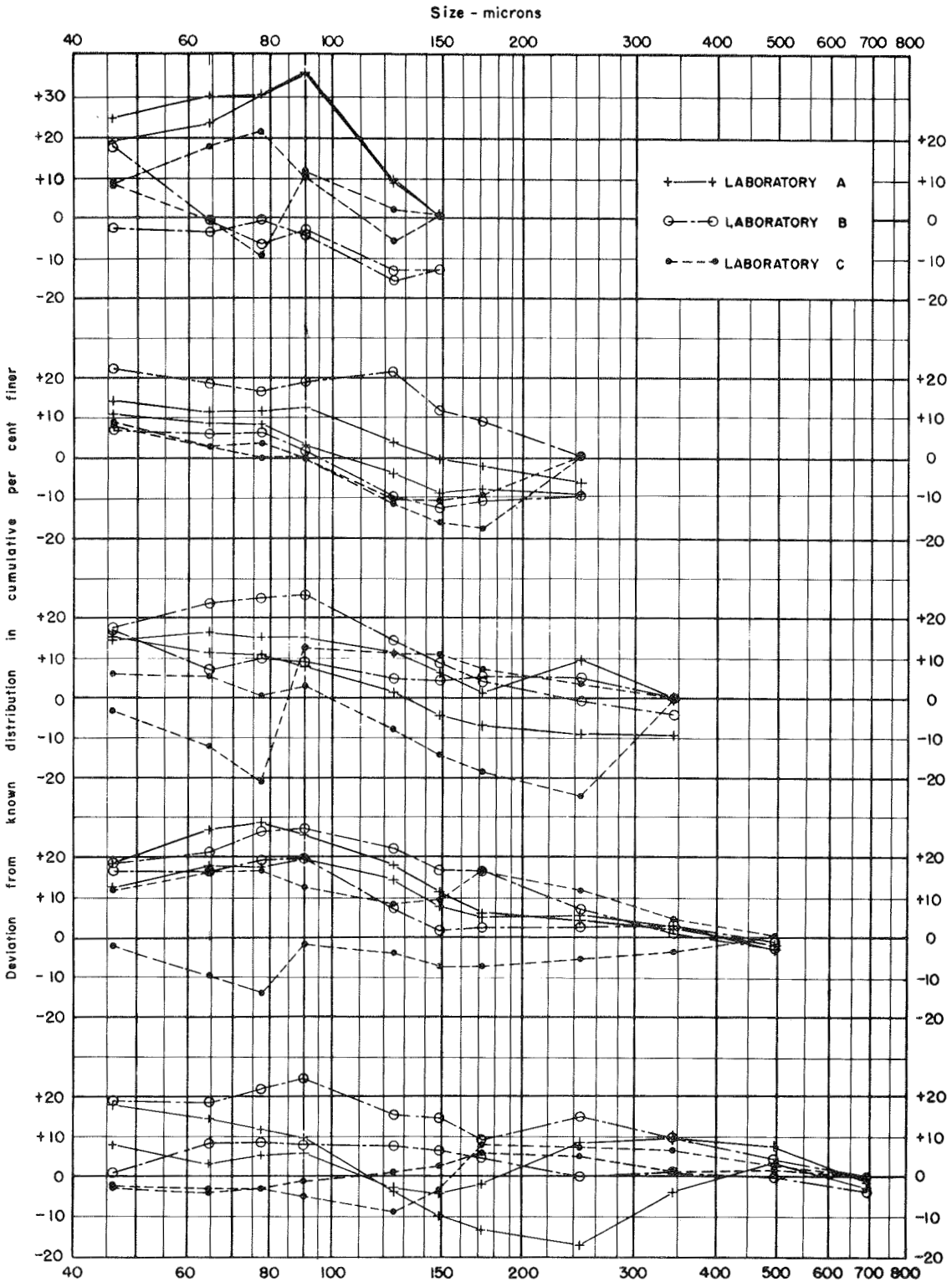


Fig. 18--Consistency of size analysis results for glass spheres, concentration 200 p.p.m.

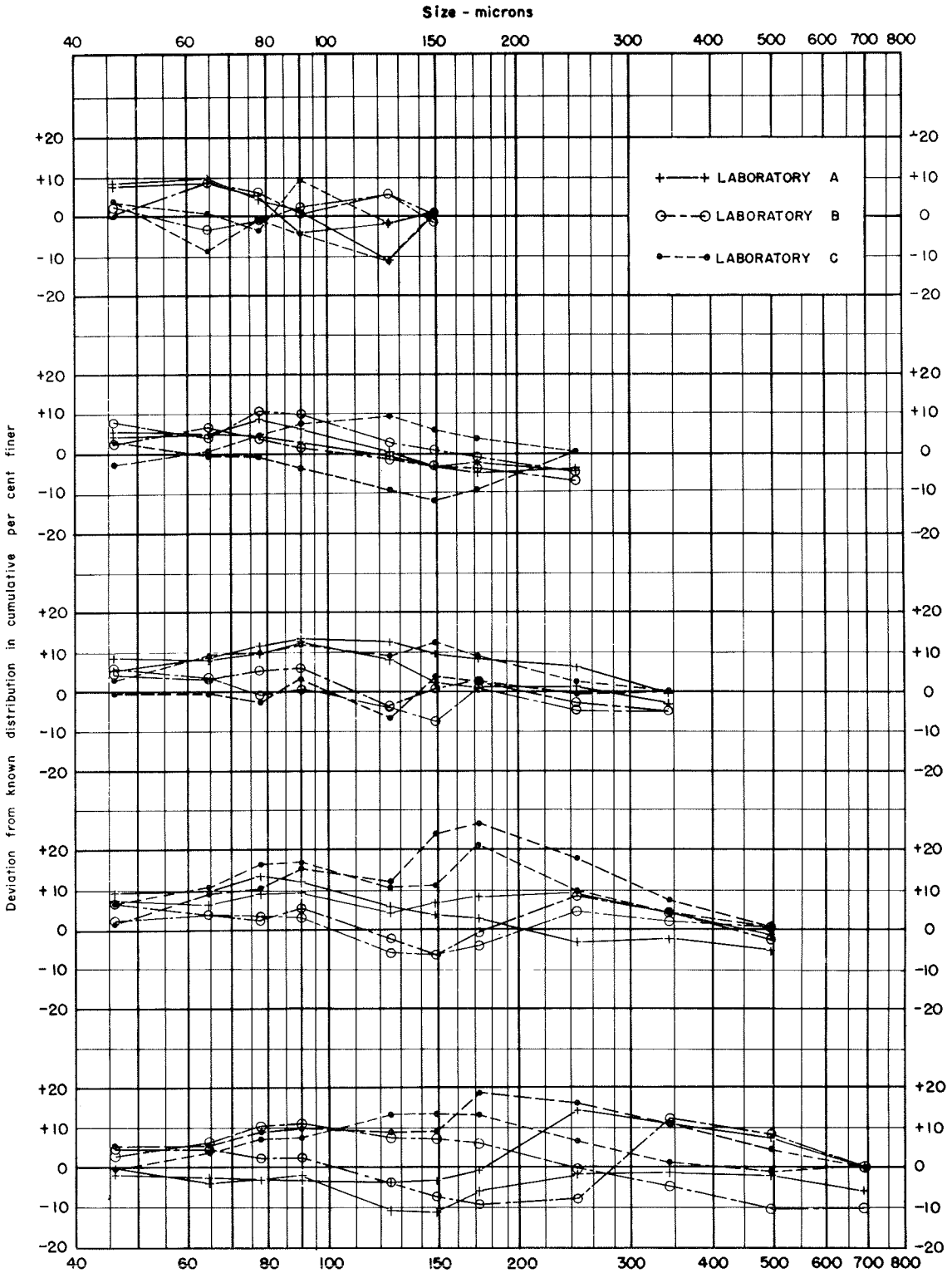


Fig. 19--Consistency of size analysis results for glass spheres, concentration 1,000 p.p.m.

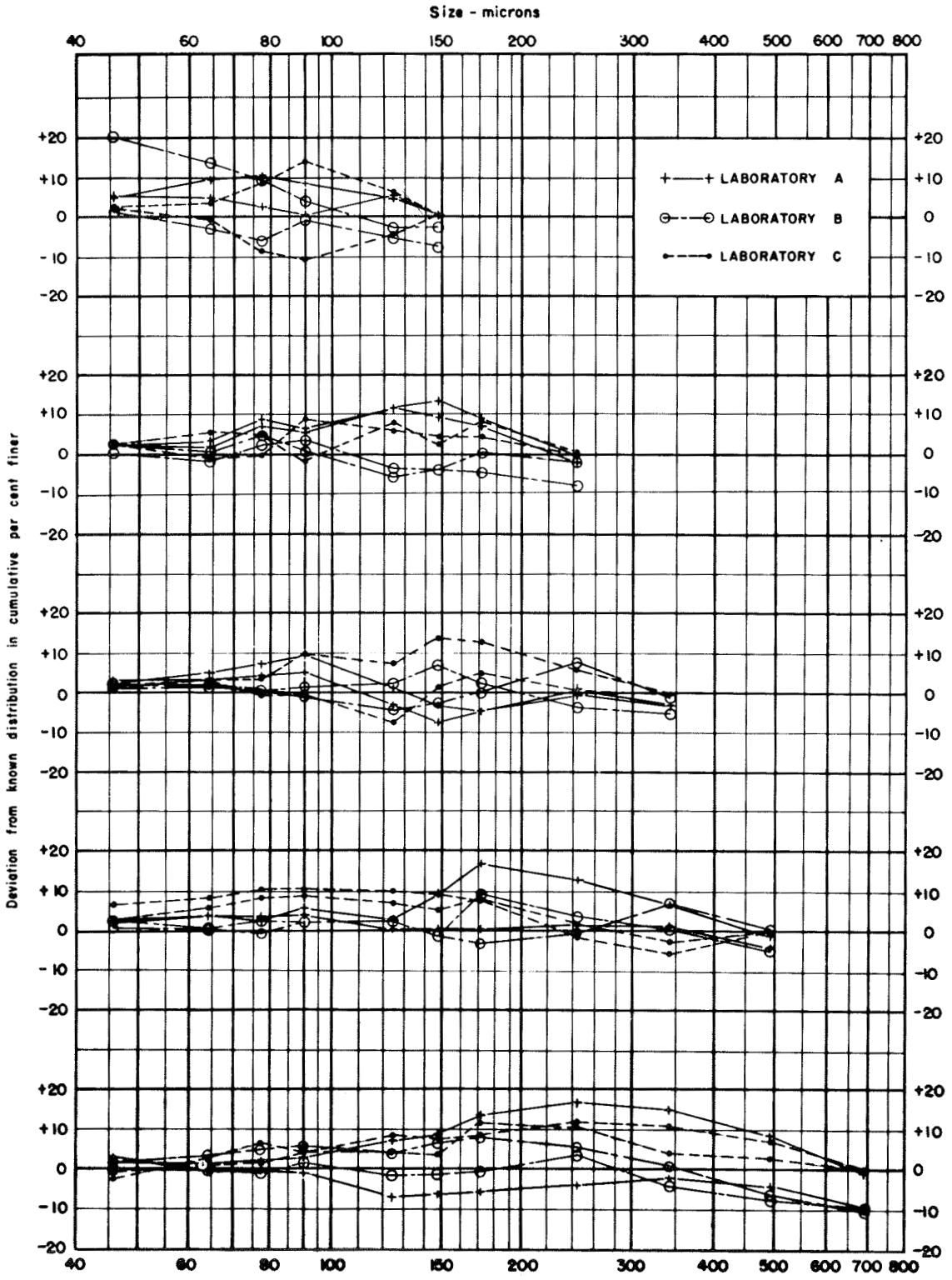


Fig. 20--Consistency of size analysis results for glass spheres, concentration 3,000 p.p.m.

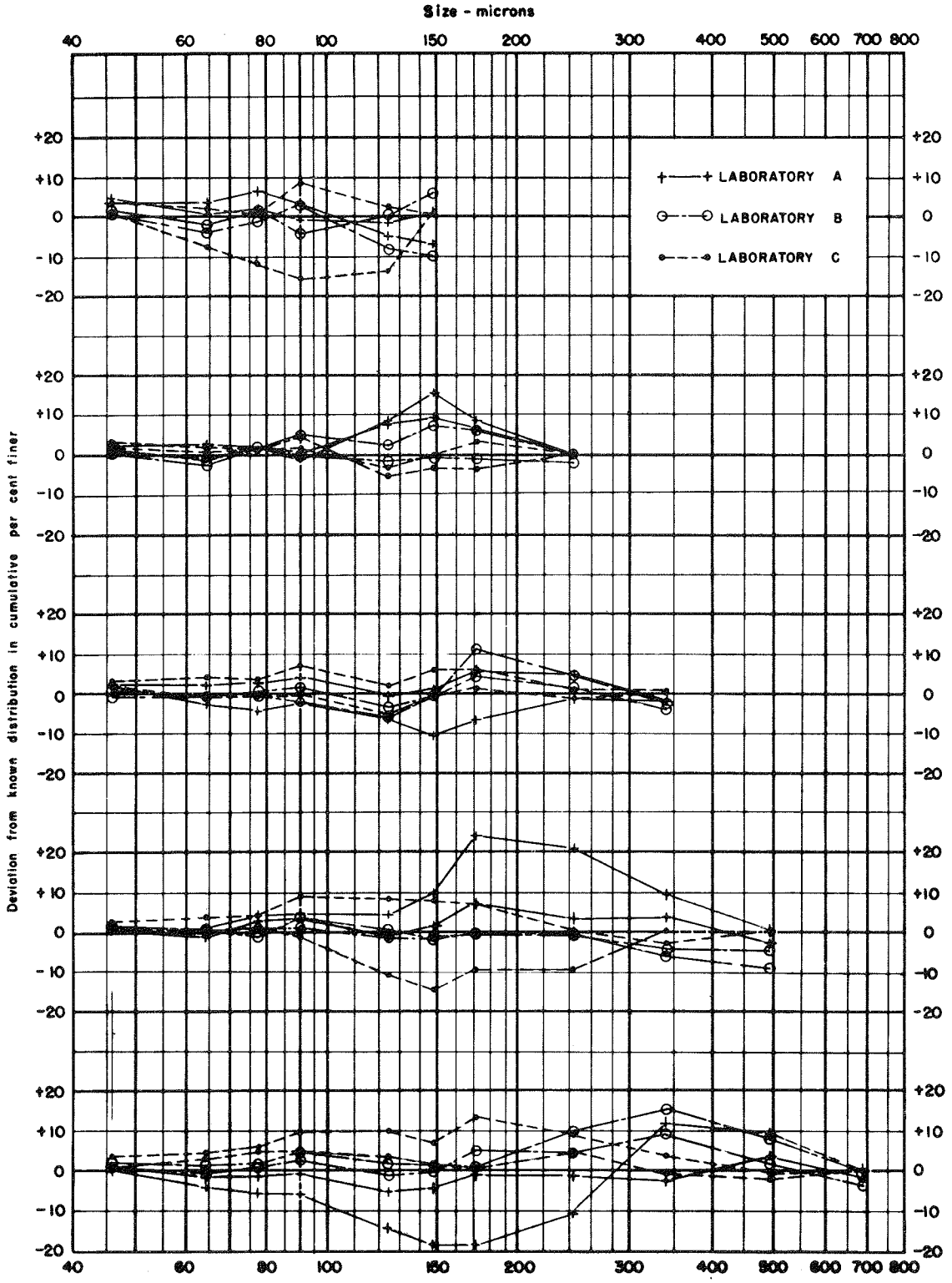


Fig. 21--Consistency of size analysis results for glass spheres, concentration 5,000 p.p.m.

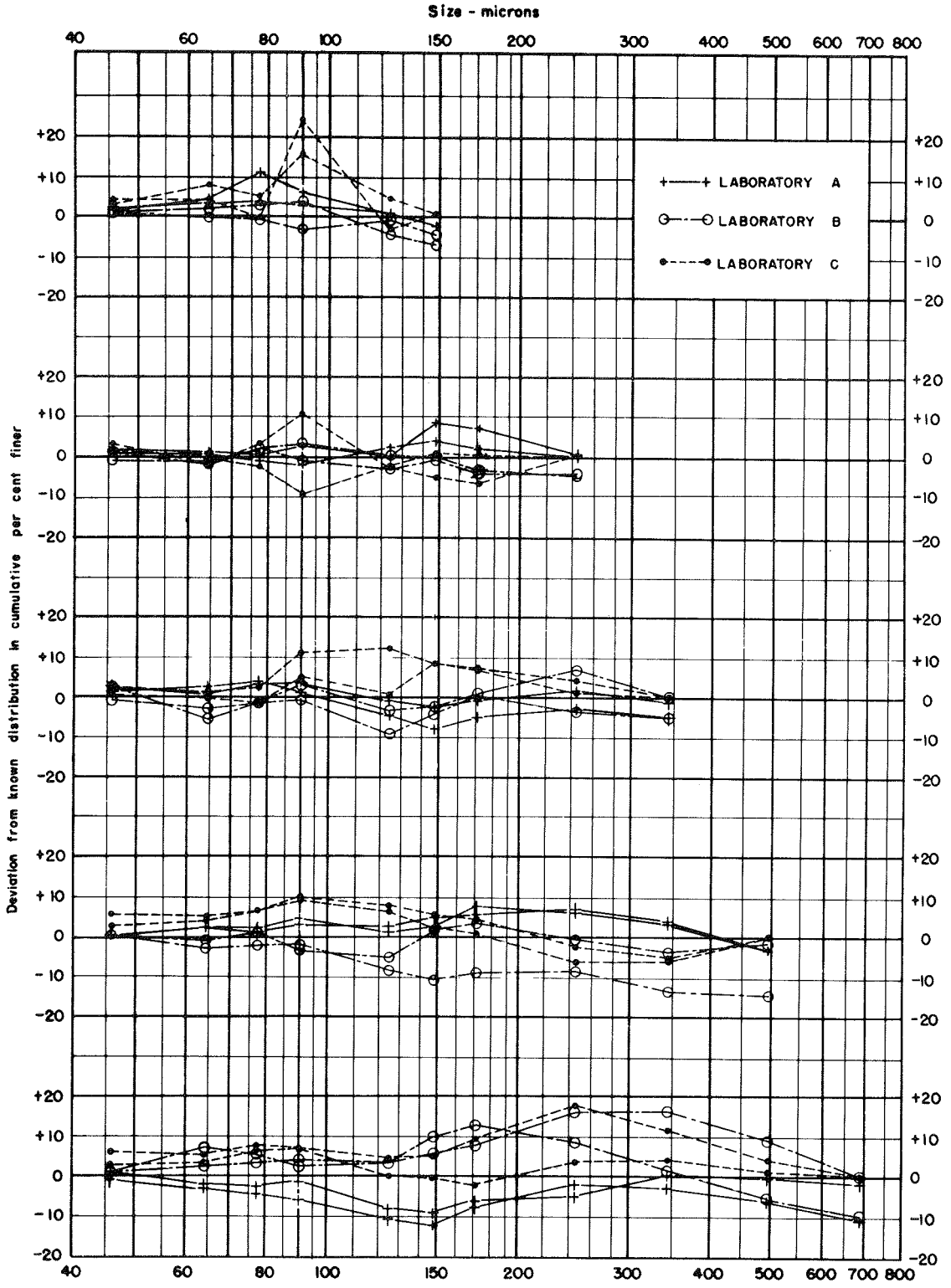


Fig. 22--Consistency of size analysis results for glass spheres, concentration 7,000 p.p.m.

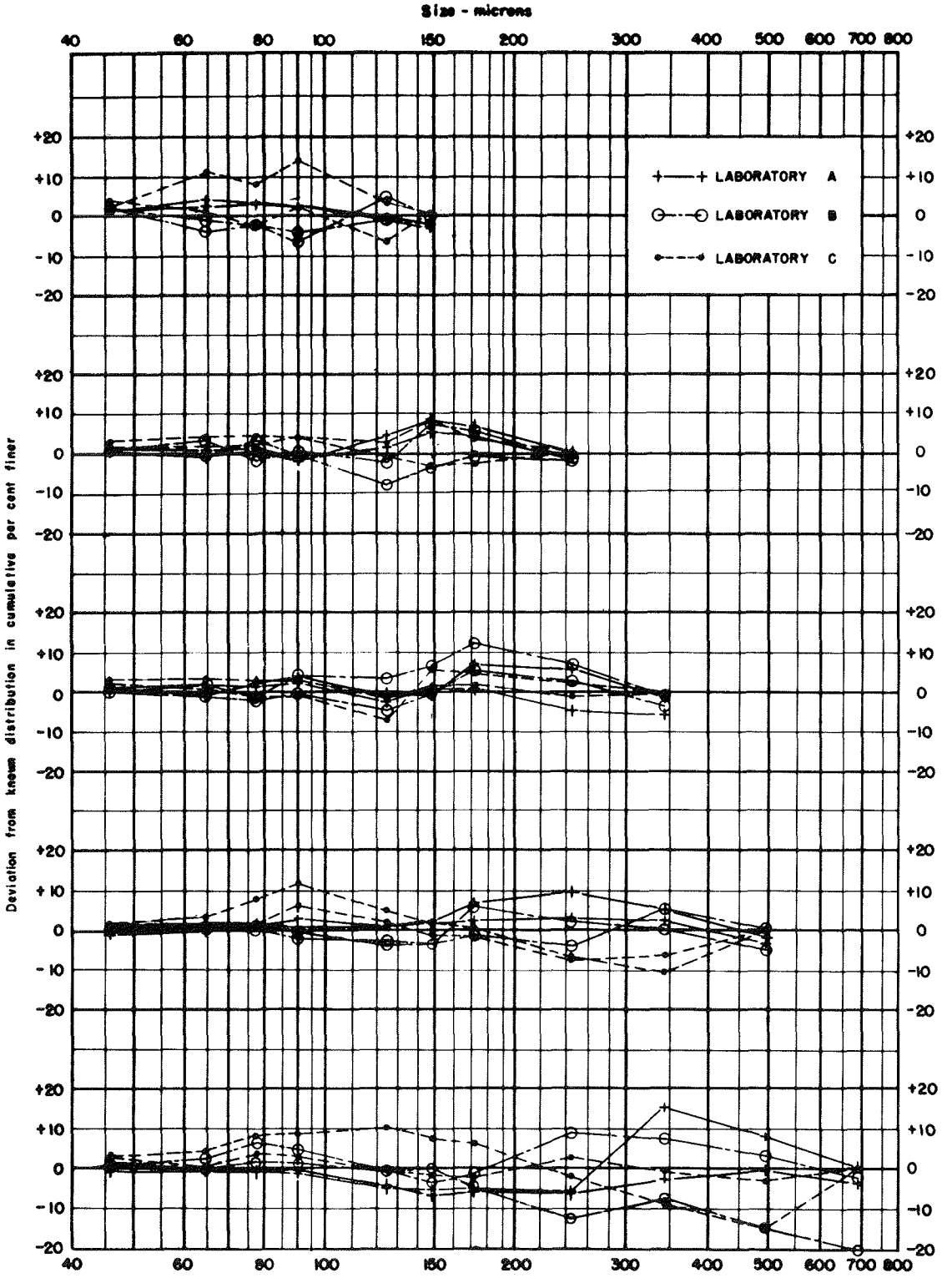


Fig. 23--Consistency of size analysis results for glass spheres, concentration 10,000 p.p.m.

TABLE 7

ACCURACY OF BOTTOM WITHDRAWAL TUBE ANALYSES OF GLASS SPHERES OF SAND SIZES
BASED ON MAXIMUM ERROR IN PER CENT FINER VALUES FOR EACH ANALYSIS

Concentration in p.p.m.	Number of Samples within Indicated Limits of Accuracy																										Per Cent of Samples within Indicated Limits of Accuracy with Respect to Concentration			
	Nominal Size Range in Microns																						With Respect to Concentration							
	20-149					20-246					20-350					20-500					20-700									
	0-5%	5-10%	10-15%	15-20%	over 20%	0-5%	5-10%	10-15%	15-20%	over 20%	0-5%	5-10%	10-15%	15-20%	over 20%	0-5%	5-10%	10-15%	15-20%	over 20%	0-5%	5-10%	10-15%	15-20%	over 20%	0-5%	5-10%	10-15%	15-20%	Over 20%	5%	10%	15%	20%
200			1	2	3			4	1	1				3	3			1	3	2		4		1	1	0	4	6	10	10	0	13	33	67
1,000		4	2				4	2				3	3				3	1		2			5	1		0	14	13	1	2	0	47	90	93
3,000		2	3		1		4	2				5	1			1	3	1	1			3	2	1		1	17	9	2	1	3	80	90	97
5,000	1	4		1		2	3		1			4	2			1	3	1		1	1	2	1	2		5	16	4	4	1	17	70	83	97
7,000	2	1	1	1	1	3	2	1			1	4	1				4	2				2	2	2		6	13	7	3	1	20	63	87	97
10,000	2	3	1			1	5				1	4	1			2	3	1			1	2	1	2		7	17	4	2	0	23	80	93	100
0-5%	5					6					2					4					2					19								
5-10%		14					18					20					16					13						81						
10-15%			8					9					8					7						11					43					
15-20%				4					2					3					4						9					22				
over 20%					5					1					3					5					1					15				
	Per Cent of Samples within Indicated Limits of Accuracy, with Respect to Size Range																																	
5%	14					17					6					11					6										11			
10%	53					67					61					56					42											56		
15%	75					92					83					75					72												79	
20%	86					97					92					86					97													92

Totals

Average Percentages

variations over 5 per cent but not over 10 per cent; variations over 10 per cent but not over 15 per cent; over 15 per cent but not over 20 per cent; and over 20 per cent. These counts are presented and summarized for each concentration and size range. This table shows the maximum error to be expected in an individual sample analysis in terms of frequency of occurrence.

The data of Table 7 show that the maximum error in an individual analysis is generally greater for the lower concentrations of sediment. Maximum errors of 15 or 20 per cent in individual analyses occur with about the same regularity in all the sizes of sediment analyzed. There are fewer analyses with maximum errors within 5 per cent when particle sizes of 350 microns and over are involved, and somewhat fewer within 10 per cent when sizes up to 700 microns are included. In regard to all analyses, Table 7 indicates that the maximum error in an individual analysis will seldom be less than 5 per cent; will be less than 10 per cent in about half of the cases; will be less than 15 per cent for three-fourths of all analyses; and will nearly always be less than 20 per cent. It should be remembered that the percentages discussed in this relation are the differences in the per cent finer figures for the test sample and those for the analysis of the sample. If 50 per cent of a given sample is finer than 149 microns and the analysis shows 60 per cent finer, then the error at 149 microns is $^{+}10$ per cent.

Table 8 presents the accuracy of the bottom withdrawal tube method of analysis on the basis of errors found in all determinations of per cent finer values. By reference to the consistency curves of Figs. 18 to 23, it will be seen that the determinations of per cent finer values

TABLE 8

ACCURACY OF BOTTOM WITHDRAWAL TUBE ANALYSES OF GLASS SPHERES OF SAND SIZES
BASED ON ALL ERRORS IN PER CENT FINER VALUES

Concentration in p.p.m.	Number of Determinations within Indicated Limits of Accuracy																									Per Cent of Determinations in Indicated Limits of Accuracy With Respect to Concentration			
	Nominal Size Range in Microns																				With Respect to Concentration							
	20-149					20-246					20-350					20-500					20-700							
	0-5 %	5-10%	10-15%	15-20%	over 20%	0-5 %	5-10%	10-15%	15-20%	over 20%	0-5 %	5-10%	10-15%	15-20%	over 20%	0-5 %	5-10%	10-15%	15-20%	over 20%	0-5 %	5-10%	10-15%	15-20%	over 20%	0-5%	5-10%	10-15%	15-20%	Over 20%	5 %	10%	15%	20%
200	12	7	5	4	8	15	15	11	5	2	18	14	10	7	5	19	12	7	15	7	34	20	4	6	2	98	68	37	37	24	37	63	77	91
1,000	23	9	4			33	13	2			30	17	7			26	20	7	4	3	32	20	12	2		144	79	32	6	3	55	84	97	99
3,000	18	13	4		1	31	14	3			43	9	2			39	16	4	1		38	21	6	1		169	73	19	2	1	64	92	99	100
5,000	26	7	2	1		39	8		1		44	8	2			45	11	2		2	47	12	4	3		201	46	10	5	2	76	94	97	99
7,000	29	4	1	1	1	43	4	1			43	9	2			38	18	4			36	22	5	3		189	57	13	4	1	72	93	98	100
10,000	30	4	2			42	6				43	10	1			50	8	2			43	18	2	3		208	46	7	3	0	79	96	99	100
0-5 %	138					203					221					217					230					1009								
5-10%		44					60					67					85					113					369							
10-15%			18					17					24					26					33					118						
15-20%				6					6					7					20					18				57					
over 20%					10					2					5					12					2				31				
	Per Cent of Determinations within Indicated Limits of Accuracy with Respect to Size Range																																
5 %	64					70					68					60					58										64			
10%	84					91					89					84					87											87		
15%	93					97					96					91					95												94	
20%	95					99					98					97					99													98

Totals

Average Percentages

were made for certain sizes of sediment regardless of the size range involved in the sample, so that 6 determinations were taken in a sample with size range 20-149 microns, while 11 determinations were used when the size range was 20-700 microns. Individual counts were made of all determinations which were within the percentage classes used for Table 7, and the data are presented in that same form.

Table 8 shows that the error in any determination may be expected to be greater for the lower concentrations of sediment. There is no definite indication of any change in accuracy in the results as the sizes of sediment in the samples change. A determination of a per cent finer value in a sample may be expected to be within 5 per cent in almost two-thirds of the cases; within 10 per cent for seven-eighths of the determinations; and nearly always within 15 or 20 per cent.

Fig. 24 has been prepared to aid in studying the average error for the total sample as a measure of the soundness of the bottom withdrawal tube method. Average errors are shown for samples having similar concentrations and covering like size ranges. In terms of average results the analyses are quite accurate, and are very accurate at the higher concentrations. This accuracy strongly supports the basic soundness of the theory and of the techniques employed. Within the range of concentrations used in these tests, the size analyses become more accurate on a percentage basis as the concentration in the sample increases; the average error decreasing from ± 5.2 per cent at a concentration of 200 p.p.m. to ± 0.5 per cent at a concentration of 10,000 p.p.m. There seems to be no definite relation between the average error of the analysis and the size of particle involved in the sample.

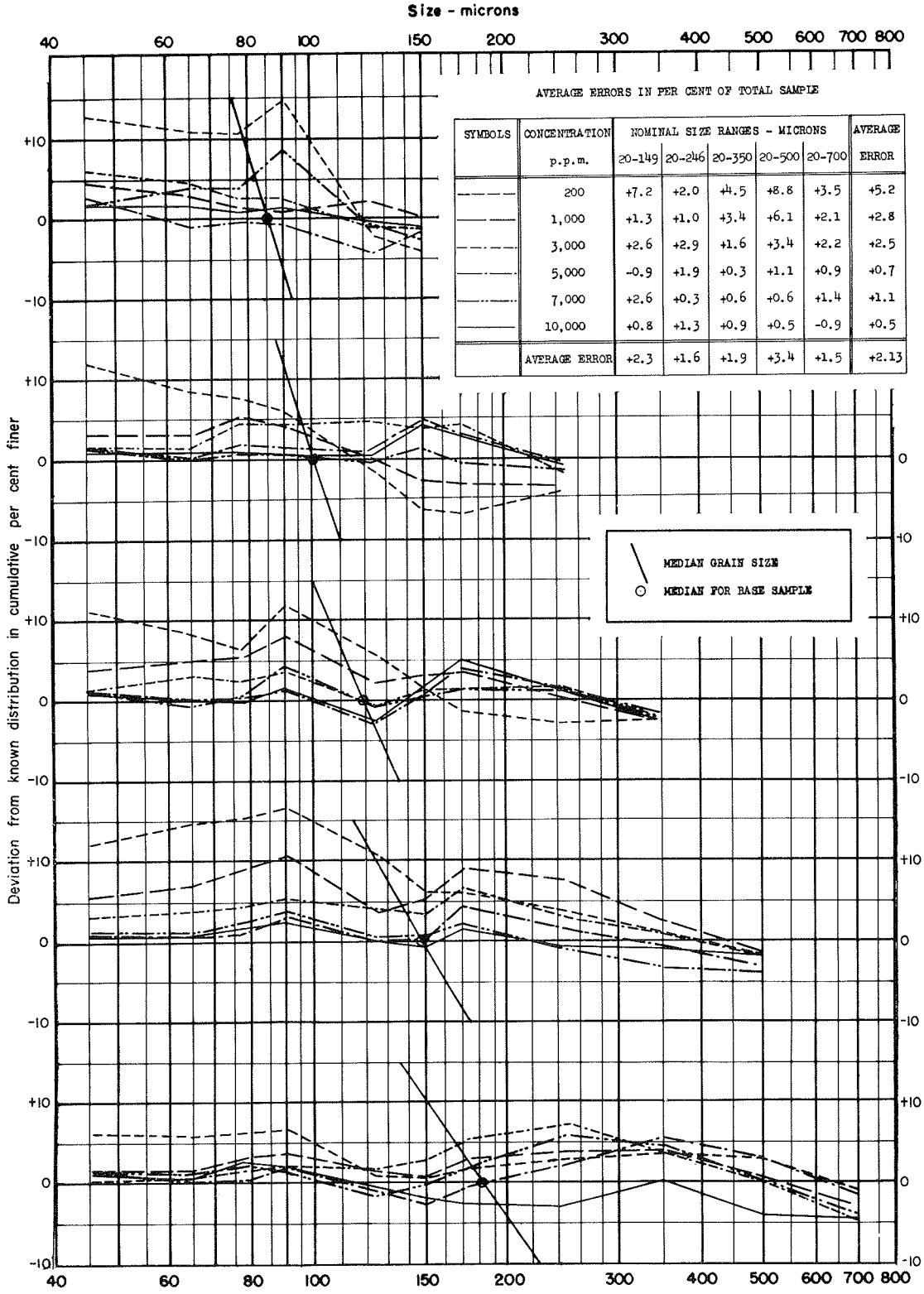


Fig. 24--Consistency of average size analysis results for glass spheres

In general, regardless of size or concentration of sediment, the results obtained with the bottom withdrawal tube method show a smaller mean grain size than that of the base samples. The median grain size for the original glass bead samples has been shown in Fig. 24. A heavy line has been extended upward and downward in such manner that the abscissa is the median grain size for a sample analysis with the error intercept as the ordinate. It is not the average error of analysis, but the error at the point on this heavy line which determines the median grain size.

Table 9 shows the median grain sizes for the original glass bead or

TABLE 9

MEDIAN GRAIN SIZE OF SAMPLES

Concentration p.p.m.	Nominal size ranges - microns				
	20-149	20-246	20-350	20-500	20-700
200	78	97	107	122	176
1,000	85	98	113	137	173
3,000	84	97	120	139	166
5,000	86	99	123	146	183
7,000	82	101	120	145	175
10,000	85	100	122	150	193
Average 200 to 10,000	83.3	98.7	117.5	139.8	177.7
Average 1,000 to 10,000	84.4	99.0	119.6	143.4	178.0
Base Sample	85.8	101	119	147	182
% Difference 200 to 10,000	-2.9	-2.3	-1.3	-4.9	-2.4
% Difference 1,000 to 10,000	-1.6	-2.0	+0.5	-2.4	-2.2

base samples, and also the median grain sizes corresponding to the average analyses presented in Fig. 24. The differences are the result of the deviations in the analyses. The median grain sizes are shown as averages for all analyses, and as averages for all analyses except those at 200 p.p.m. The percentages by which the averages differ from the median sizes for the base samples are also shown. The median grain sizes based on the average analyses are satisfactorily accurate.

The preceding discussion of the size analyses was in terms of per cent finer values and median grain size, and, consequently, did not show the degree of accuracy with which the analyses indicate the proper amount of material in each size class or fraction. Evaluation of the accuracy of analyses in various size classes has been made the subject of a second study of the results of the bottom withdrawal tube method of analysis of glass bead samples. In order to simplify this problem, the fractions studied are divided as follows: Smaller than 62.5 microns, 62.5 to 125 microns, 125 to 250 microns, 250 to 500 microns, and 500 to 700 microns. These are nominal size classes; the actual size ranges determined differ by a very few microns from these values, but the differences would have no significant effect on the results. The quantity of material in each of these size classes was first determined as a percentage of the total for each of the base samples. The basic distribution for each type of sample is shown in Table 4. Then the quantity of material in each size class was computed from the bottom withdrawal tube analysis. The difference between the quantity compounded in the sample and that shown by the analysis was expressed as a percentage of the quantity in the original sample. These percentage errors have

been used as the basis for the study of the accuracy of analysis in each size class.

A tabulation of the errors in terms of concentration and size class is presented in Table 10. Errors are divided into the following groups: 0 to 10 per cent, 11 to 20 per cent, 21 to 40 per cent, 41 to 80 per cent, and those over 80 per cent. The accuracy of the results increased with concentration. For the concentration of 200 p.p.m. the accuracy was relatively poor. However, the reduction in accuracy at the lower concentrations was mainly in the size classes under 250 microns, and was most obvious for the size class containing particles smaller than 62.5 microns. The average percentages shown are those derived from the totals, and are also the averages of the percentages within the given limits with respect to concentration. The averages of the percentages with respect to size classes would give somewhat different figures, because radically different numbers of items were included in the various size classes. The averages show that generally over half of the determinations of the quantities in the various size classes were within 20 per cent of the base samples, three-fourths of the determinations were within 40 per cent, and all but 6 per cent were within 80 per cent. With respect to size classes the percentages show increasing accuracy with a decrease in size of the particle down to 62.5 microns, but with a decrease in accuracy for the smallest size class. As an indication of what actually happened in these tests, or of what might be expected of tests covering similar size distributions, these are representative data. However, this table does not show the quantity of the sample contained in the various size classes and that is a controlling factor in

TABLE 10

ACCURACY OF BOTTOM WITHDRAWAL TUBE ANALYSES OF GLASS SPHERES OF SAND SIZES

BASED ON ERRORS IN EACH SIZE CLASS

Concentration in p.p.m.	Number of Determinations within Indicated Limits of Accuracy																									Per Cent of Samples within Indicated Limits of Accuracy With Respect to Concentration			
	Nominal Size Range in Microns																					With Respect to Concentration							
	20-62.5					62.5-125					125-250					250-500					500-700									
	0-10%	11-20%	21-40%	41-80%	over 80%	0-10%	11-20%	21-40%	41-80%	over 80%	0-10%	11-20%	21-40%	41-80%	over 80%	0-10%	11-20%	21-40%	41-80%	over 80%	0-10%	11-20%	21-40%	41-80%	over 80%	0-10%	11-20%	21-40%	41-80%	Over 80%	10%	20%	40%	80%
200	2	1	5	5	17	6	8	14	2		5	4	10	7	4	3	3	5	3	4	2		2	2		18	16	36	19	25	16	30	61	78
1,000	4	1	9	12	4	14	10	5	1		6	11	8	3	2	1	4	7	6			1	1	3	1	25	27	30	25	7	22	46	72	94
3,000	7	8	8	6	1	19	6	5			8	7	8	7		7	3	3	5				1	3	2	41	24	25	21	3	36	57	79	97
5,000	13	10	5	2		20	7	2	1		10	9	7	2	2	6	5	2	4	1	1	1	2	1	1	50	32	18	10	4	44	72	88	96
7,000	10	4	10	5	1	20	6	4			12	8	7	3		3	4	7	4		2			3	1	47	22	28	15	2	41	61	85	98
10,000	14	8	6	2		19	9	2			13	5	10	2		2	3	7	6		1		2	1	2	49	25	27	11	2	43	65	89	98
0-10%	50					98					54					22					6					230								
11-20%		32					46					44					22					2					146							
21-40%			43					32					50					31					8					164						
41-80%				32					4					24					28					13					101				
over 80%					23										8					5					7					43			
	Per Cent of Determinations within Indicated Limits of Accuracy with Respect to Size Classes																																
10%	28					54					30					20					17										34			
20%	46					80					54					41					22											55		
40%	69					98					82					69					44												79	
80%	87					100					96					95					81													94

Totals

Average Percentages

the apparent accuracy of an analysis in the different size fractions.

The percentage errors in the analyses of the quantities of material in the size classes are retabulated in Table 11. In this case, the errors are listed with respect to the size range of beads contained in the sample, and the percentage of the total sample included in each of the size classes is given at the bottom of the table to provide a means of making a more revealing comparison of the relation between errors and per cent of material in the size class. As would be expected, the errors varied sharply (in many cases almost inversely) with the percentage of the sample contained in the size class. If 10 per cent of a sample was contained in the coarsest size fraction, then about one time in six the results of the analyses showed over 80 per cent error in the coarsest size class. Because the average error in the over 80 per cent group was around 100 per cent, it means that if the sample was 90 per cent finer than this coarsest size, one time in six the analysis showed either about 100 or 80 per cent finer than this size. If the coarsest fraction contained 20 per cent of the total material in the sample the percentage errors were reduced to about half of what they were when only 10 per cent was in that class. For samples containing equal percentages of material, the results were better for the classes composed of the smaller sizes of particles. The results were also relatively better for samples in the narrower ranges of particle size.

TABLE 11

ACCURACY OF BOTTOM WITHDRAWAL TUBE ANALYSES OF GLASS SPHERES OF SAND SIZES

ERRORS IN EACH SIZE CLASS TABULATED BY SIZE RANGE IN SAMPLES

Nominal Size Range of Samples in Microns	Number of Determinations within Indicated Limits of Accuracy																								
	Nominal Size Range in Microns																								
	20-62.5					62.5-125					125-250					250-500					500-700				
	0-10%	11-20%	21-40%	41-80%	over 80%	0-10%	11-20%	21-40%	41-80%	over 80%	0-10%	11-20%	21-40%	41-80%	over 80%	0-10%	11-20%	21-40%	41-80%	over 80%	0-10%	11-20%	21-40%	41-80%	over 80%
20-149	11	10	6	6	3	25	6	5			4	6	5	13	8										
20-246	16	6	9	4	1	21	11	4			16	7	9	4											
20-350	8	8	10	6	4	19	11	5	1		13	11	10	2		5	8	8	12	3					
20-500	10	3	6	6	11	20	10	6			12	9	14	1		10	7	12	6	1					
20-700	5	5	12	10	4	13	8	12	3		9	11	12	4		7	7	11	10	1	6	2	8	13	7
Totals	50	32	43	32	23	98	46	32	4		54	44	50	24	8	22	22	31	28	5	6	2	8	13	7
	Per Cent of Determinations within Indicated Limits of Accuracy																								
Limits	10%	20%	40%	80%		10%	20%	40%	80%		10%	20%	40%	80%		10%	20%	40%	80%		10%	20%	40%	80%	
20-149	31	58	75	92		69	86	100	100		11	28	42	78											
20-246	44	61	86	97		58	89	100	100		44	64	89	100											
20-350	22	44	72	89		53	83	97	100		36	67	94	100		14	36	58	92						
20-500	28	36	53	69		56	83	100	100		33	58	97	100		28	47	81	97						
20-700	14	28	61	89		36	58	92	100		25	56	89	100		19	39	69	97		17	22	44	81	
Average	28	46	69	87		54	80	98	100		30	54	82	96		20	41	69	95		17	22	44	81	
	Per Cent of Total Sample in Each Size Fraction																								
20-149	21.5					69.7					8.8														
20-246	14.3					57.0					28.7														
20-350	11.4					42.4					36.3					9.9									
20-500	8.6					31.2					37.9					22.3									
20-700	8.1					23.1					33.5					25.6					9.7				

IV. GENERAL DISCUSSION

21. Effect of operational techniques on accuracy of analysis--The results of the tests on the bottom withdrawal tube do not differentiate between the errors inherent in the apparatus and those resulting from operational technique. The sum of the errors which may be attributed to these two sources are usually relatively small in comparison to the total error of analysis. However, serious errors may be introduced in the first and second withdrawals due to inadequate operational technique, especially if the particle sizes are relatively coarse. A few of the operational techniques will be considered in terms of errors which may be present, even though the magnitude of the errors cannot be evaluated within the scope of this report.

a. Dispersion--The derivation of the Odén curve of Sections 14 and 15 assumed that particles of all sizes were uniformly dispersed at the beginning of sedimentation. As this is the unique curve that will yield the known cumulative size distribution for the material, it follows that perfect dispersion is fundamental to the bottom withdrawal tube method.

Following the "Detailed Test Procedure for the Bottom Withdrawal Tube" of Appendix A, Report No. 7, it is necessary to distribute the coarser particles along the tube before beginning the tube inversions. This is accomplished by holding the tube nearly horizontal but with the nozzle end slightly raised, and shaking the particles out along the tube until their distribution is fairly good. When this process is complete, the tube should still be in an inclined position with the bubble at the nozzle end. The tube is then placed in the upright position at the start of the repeated inversions by which the sediment is to be dispersed in the tube. As soon as the bubble reaches the end of the tube, the tube is again inverted, and the process continued until dispersion is considered to be complete. Then with the bubble at the bottom of the tube, timing is begun, and the tube is fixed in an upright position for the start of the settling process.

The period of time between tube inversions is dependent only on the speed of the bubble which must travel the full length of the

tube to insure agitation of all the material. This requires about 5 sec. For the moment, neglect the transporting and dispersing effects of the bubble and consider only the motion of the particles due to their weight. Then a simple analysis of the mechanics of the dispersion may be made to show one avoidable cause of poor initial distribution.

Particles of a certain size fall with a velocity of $100/t_o$ cm. per sec. when t_o is the time in seconds for that size of particle to fall 100 cm. As the tube is righted prior to the repeated inversions, $5/t_o$ of this size settles out of suspension in the 5 sec. required for the bubble to travel the 100 cm. length of water column in the tube. After the first inversion, the settled portion falls a distance $500/t_o$ away from the nozzle, and when the tube is righted after any future inversion this portion, $5/t_o$ of the given size, will start falling from a point $500/t_o$ cm. above the nozzle. See Condition I of Fig. 25. If the time for the first withdrawal is longer than the 5 sec. of bubble travel, the concentrations of $5/t$ of each size will drop into the first withdrawal. For example, if a time of 7 sec. is used for the first withdrawal, all material having a fall velocity of t and starting from any point less than $700/t_o$ above the top of the first withdrawal will be included in that withdrawal. After the first withdrawal, the material remaining in suspension will be distributed with the same intensity and at the same position as it would occupy with perfect dispersion. Therefore, for the bottom withdrawal tube analysis, this type of dispersion should give the same results as perfect dispersion, and so should be completely satisfactory.

On the other hand, if the operator were to start the inversions by turning the nozzle end of the tube upward immediately after distributing the coarser particles by eye, conditions in the suspension would be entirely different. Now, the $5/t_o$ portion of each size will settle out at the wide mouth end of the tube, and each succeeding time that the tube is righted this material will start falling from the top of the tube. See Condition II of Fig. 25. It is obvious that these concentrations of material cannot reach the bottom of the tube in the same time as in a perfect dispersion in which the sizes would be distributed uniformly upward from the nozzle to the top of the tube. Those particles bunched near the top of the tube should be distributed through the lower $500/t_o$ cm. of the suspension.

To indicate the errors possible from the assumed Condition II, a few points of the resulting Odén curve are shown in Fig. 26 and compared with the curve for uniform dispersion. The computations were based on samples with the 20 to 700 micron size range as shown in Table 4. The size distribution determined by the errant curve would show too fine a material.

The actual dispersion in the bottom withdrawal tube would not

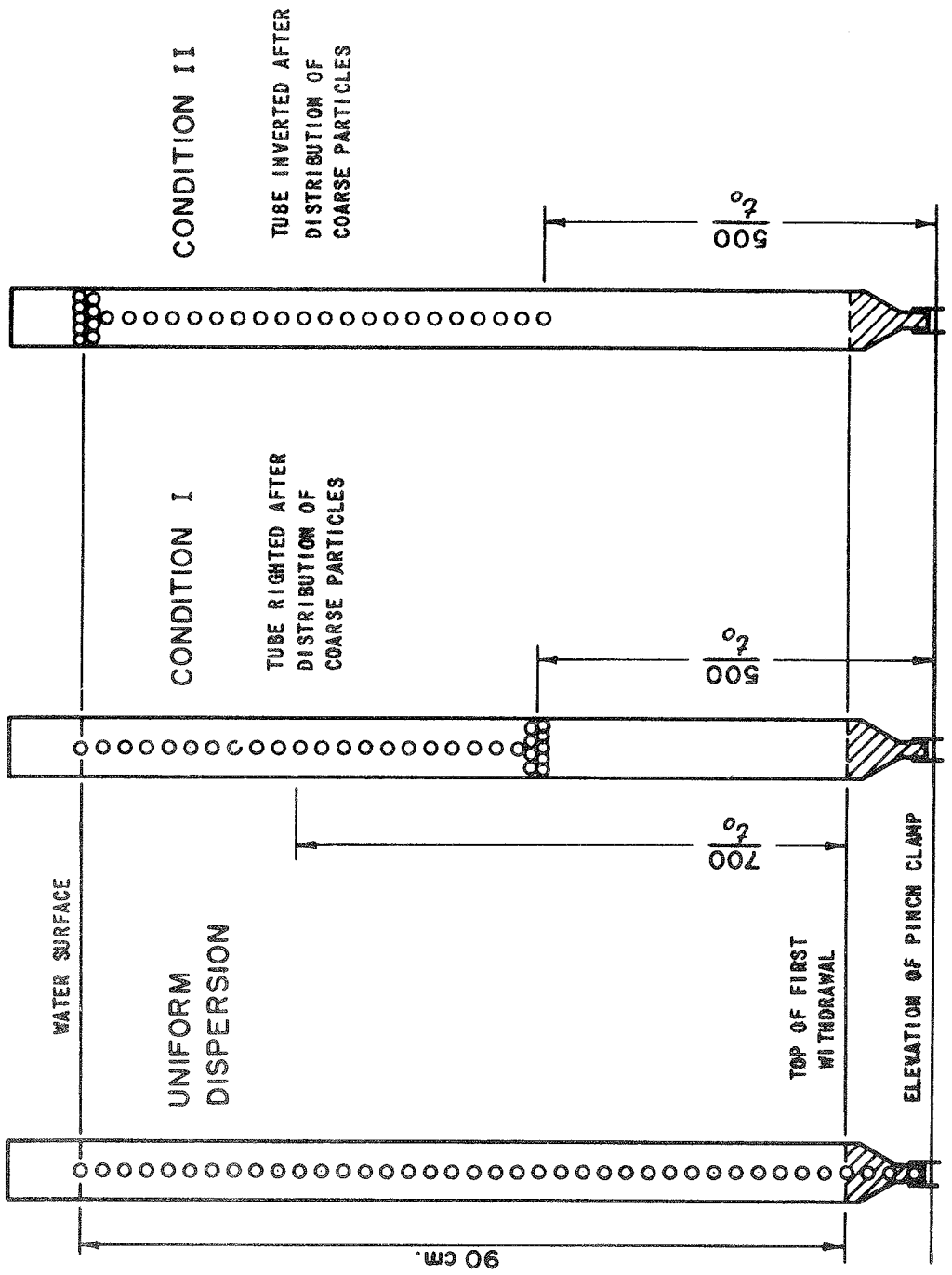


Fig. 25--Initial distributions of particles obtained under assumed conditions of mechanical dispersion

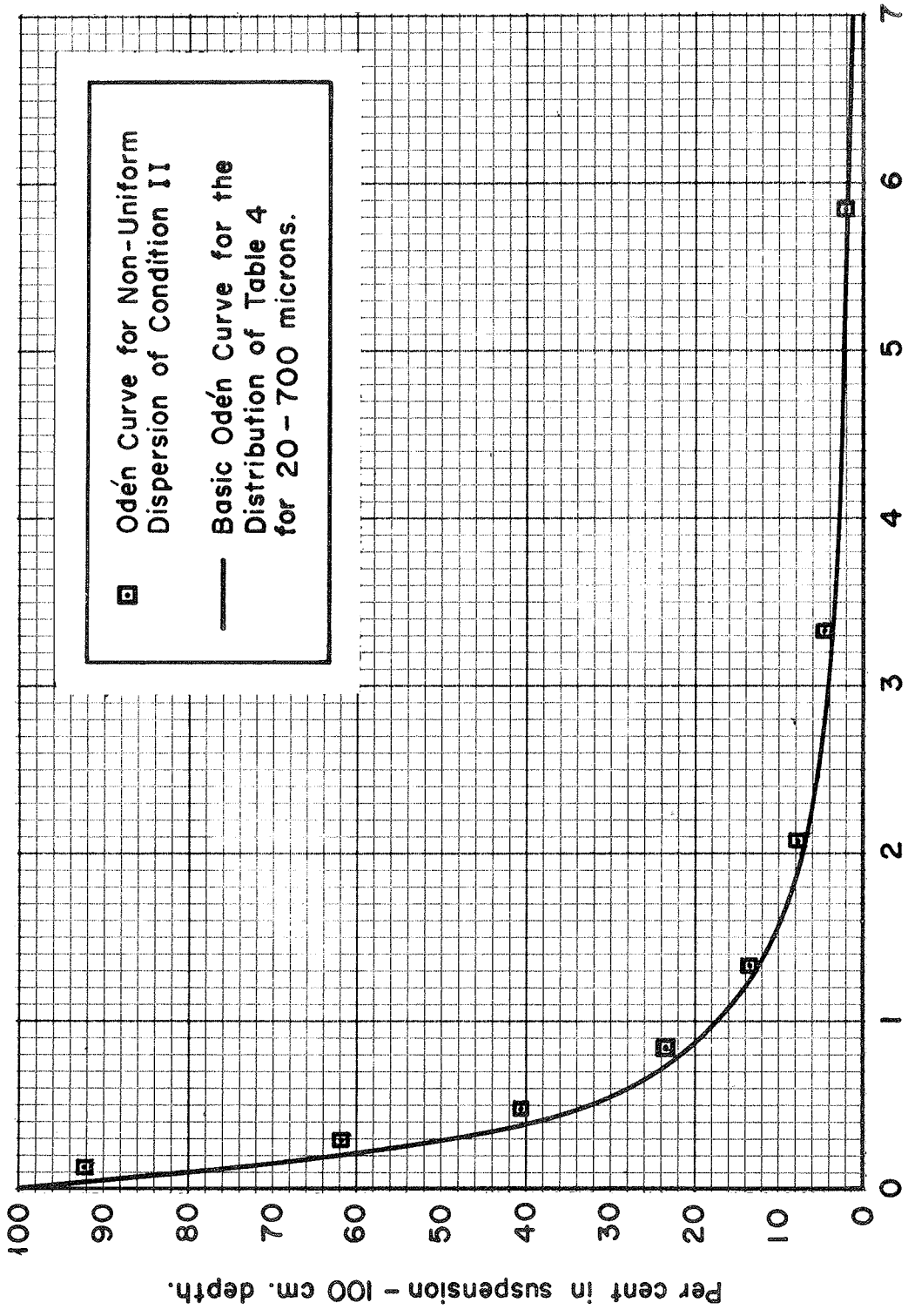


Fig. 26--Effect on Odén curve coordinates of the incomplete dispersion of Condition II

be as erroneous because the dispersing and transporting actions of the air bubble have been neglected in this analysis, which has been made only to emphasize the possible effects of a small change in technique of dispersion.

b. Bubble travel--For 5 sec. after sedimentation has started in the bottom withdrawal tube, a large air bubble moves upward through the suspension almost entirely filling the cross section, extending over nearly 1/5 of the height of the water column, with a wake of strong eddies and fine bubbles trailing behind it. In this glass bead investigation the first withdrawal was usually made only 2 sec. after the bubble broke at the top of the column and, therefore, it would not be reasonable to expect the sedimented material to have fallen in complete accord with the fall times for dispersed material in still water. In general, one might expect settling to be hindered at least during the first 7 sec. Particles may be carried upward by the surface tension of the air water boundaries of the bubbles and by the water moving upward into the area of decreased pressure just below the large bubble. The vertical components of the turbulent velocities induced by the repeated inversions may have a minor effect on settling velocities. If there are significant fluctuations continuing for some time after the tube is righted, their net effect will be to transport material upward from regions of higher concentration to regions of lower concentrations, thus delaying settlement.

c. Particles sliding along walls--Because of the method of dispersion in the tube, the coarsest grains tend to collect along one side of the tube. These particles have a tendency to slide along the side during and immediately after the dispersion process. The effect on the fall velocity of the coarse particles is obvious. Even though the passage of the bubble may throw them into the interior of the suspension, settling has been in progress at a reduced rate for some time. If the time of the first withdrawal is great enough the coarsest particles may be entirely included, although their fall velocities have been slightly reduced.

d. Time of withdrawal--According to the instructions in Report No. 7, time of settling is to begin at the moment the tube is uprighted for the last time, the bubble being at the bottom of the tube. For uniformity of procedure and to conform with theoretical concepts of dispersed sedimentation, it is strongly recommended that this practice be continued. Some operators have tried to compensate for the possible reduced rate of fall of the particles during the first moments of sedimentation by starting the timing at some instant after the bubble starts upward. The results as plotted in Figs. 13 to 17 indicate that a reduction in time of 4 sec. for the first withdrawal, and perhaps 2 sec. for the second would check the basic Odén curves better. That is to say, that if the timing of settling had been arbitrarily started when the bubble reached the halfway mark in its final upward trip, this correction

would have given generally improved results. Such a correction would be both arbitrary and approximate, and presumably not truly representative of any specific withdrawal or sample. Incidentally from Figs. 13 to 17, a correction of 4 sec. seems definitely too great except for the first withdrawals.

If a time correction is to be made to early withdrawals--and a small correction is possibly justified--it should be made to those times determined by standard procedure, and made in such a way as to be clearly obvious and readily subject to later review and revision if necessary.

Careful timing is mandatory, particularly during the first withdrawals of a coarse sample, when a small length of time means a large change in the amount of material remaining in suspension. This early portion of the Odén curve is subject to large errors in graphical analysis because of the small angle between the tangent to the curve and the per cent in suspension axis. Any small change in shape of the curve in this region results in a magnified difference in size distribution. The operator can improve the definition of the shape of the settling curve by judicious spacing of withdrawals with respect to time.

It should be emphasized that the time of withdrawal is taken at the end of each withdrawal. The objective of the experimental procedure is to determine the per cent of material remaining in suspension at a given time. Only at the instant that the valve is closed can the operator be certain that all particles still in the tube are in suspension, but if care has been exercised to take a fast enough withdrawal to insure a complete flushing of material from the lower end of the tube, then the remaining particles must be in suspension at that instant.

22. Effect of apparatus on accuracy of analysis--The sum of the errors which may be attributed to the influence of operational techniques plus limitations of apparatus are in general relatively small. They may reach serious proportions for the first and second withdrawals, and throughout the analysis of samples of low concentrations, and these more critical errors may be derived in part from inadequate apparatus. Some of the sources of inaccuracies will be discussed although the errors cannot be evaluated quantitatively.

a. Wall effects--The ideal condition for determining the fall velocities of particles, either individually or in mass,

would be within a fluid of limitless extent. The proximity of the walls forming the boundary of the suspension have a slowing effect on the fall velocities of adjacent particles. From geometry, it can be shown that one-half of the area of a circular cross section lies within 3/10 of the radius from the wall. For the case of uniform dispersion in the 25 mm. tube, one-half of all particles coarser than 350 microns fall inside of a region within 10-grain diameters from the wall, which implies the possibility of reductions in fall velocity because of wall effects [4].

The nozzle at the base of the bottom withdrawal tube has only eight per cent of the area of the main tube so that nearly all particles settling out of a well dispersed suspension will land on the sloping walls of the contraction. Consequently, the operator must obtain a positive flushing action with each withdrawal so every particle which has settled on the shoulders or in the bottom of the tube will be withdrawn. If this is done the contracted section should have no effect on the quantity of material settled out.

b. Fall velocity of particles in a dispersed system--The fall times of Table 3, determined by timing the fall of individual particles in still water, are applied without correction to the settling of particles in a dispersed system. The bottom withdrawal tube theory assumes a suspension so dilute that particles fall without mutual interference. The highest concentration used in this investigation, 10,000 p.p.m. by weight, means only 1 per cent by weight or 0.35 per cent by volume.

The effective fluid cross section of the tube is reduced by the volume of the solid particles. The density is increased because of the weight of the particles which are heavier than water. The viscosity is increased by the addition of the particles. However, in the dilute suspension considered here, the changes would be negligible for the size of particle involved. If clay and fine silt particles had been present in large quantities, the effect on viscosity might be significant [7].

The available results of fundamental research on this problem are from two sources. Medical physicists have investigated the fall velocities of particles smaller than 4 microns and found that an increase in the concentration in a dispersed suspension resulted in reduced velocities of fall. Chemists interested in sizes coarser than fine silt and in concentrations over 10 per cent found that for high concentrations of these sizes, the effect of concentration was to decrease the fall velocity of the particles in suspension [8].

Previous discussion of the influence of apparatus and operational techniques has treated exclusively of those factors which would indicate that the fall velocities of the particles as obtained by the bottom withdrawal tube method would be too low. If

however, it is assumed that the dispersion of particles is not uniform throughout the tube, there is a possibility that density currents may form in the suspension. Because the higher concentrations will tend to produce the higher densities, these regions of higher concentrations will be included in the downward currents, while the upward currents will be areas of lighter concentrations which have been displaced upward. The net effect will be to accelerate the average fall velocity of the material.

It is generally reported by those experienced in size analysis work that the presence of coarse particles within a suspension will tend to increase the fall velocities of finer particles included in the same settling process.

When a sediment of the coarse silt or sand sizes is introduced at the top of a column of water, the material falls considerably faster than would be expected from the fall velocities of the individual component particles. This has been proven by extensive tests with glass beads in connection with the development of a visual settling tube method of size analysis which is now under study. This condition is not analogous to that of a dispersed system, but is mentioned only for its value as a comparison which possibly bears on the fall velocity problem under consideration.

c. Weight of withdrawals--In this series of bottom withdrawal tube tests with glass beads, the weight of the original sample was known. For samples with concentrations of 200 p.p.m. the sum of the weights of the withdrawals generally exceeded the weight of the sample by a considerable percentage. Table 10 shows that the decrease in accuracy accompanying the decrease in concentration is almost entirely in the finer sizes of material. That is, the errors increased in that portion of the Odén curve which is based on withdrawals containing minute quantities of material and where any errors would be magnified by the depth factor. It has been concluded that erroneous weights of the material in the withdrawals is the largest factor in creating the errors which are so prominent at concentrations of 200 p.p.m. and which still show up somewhat at higher concentrations.

The errors in weights of the withdrawals cannot be attributed to the balances used, except possibly to a minor degree. In laboratory "B" the dusty, and sometimes extremely dusty, condition of the air is believed to have contaminated the withdrawals to a considerable extent. There is some evidence that at times the distilled water contained sufficient dissolved solids to introduce slight errors. The extremely high humidity of the air in the laboratory was perhaps the most serious contributory factor in the excessive weights of withdrawals.

d. Method of cleaning the bottom withdrawal tube--Glass bead samples Nos. 49 to 52 and 56 to 63, made up as listed in Table 4,

were analyzed by laboratory "A" but complete results were not obtained. Substitute samples were compounded and analyzed in laboratory "B" and the results used in lieu of the incomplete series. In making these later tests a more effective method of cleaning the bottom withdrawal tube was used.

The bottom withdrawal tube [9] was cleaned just prior to the processing of each sample, and the time of cleaning appears to be important. A cleaning agent was compounded by adding 1 liter of concentrated sulphuric acid to a 35 ml. saturated sodium dichromate (technical) solution. A small quantity of this cleaning solution was rinsed around in the tube until all of the area was covered by a film of cleaner. The cleaner was then drained out and the tube was flushed with tap water and again with distilled water. The cleaner may be reused. This cleaner is for glassware only, and contact with the skin or clothing should be avoided.

Generally when glass bead samples were analyzed in the bottom withdrawal tube, it was noticed that rings of beads or individual beads remained adhering to the sides of the tube. After the last withdrawal had been made, the tube was rinsed out and the rinse was dried to obtain the weight of the beads which had been left as a residue in the tube. Comparative figures on the weight of beads in the rinse are shown in Table 12 for the series of samples in which

TABLE 12

WEIGHT OF GLASS BEAD RESIDUE REMAINING IN TUBE

Sample weight	Residue					
	Laboratory B		Laboratory C		Special cleaning	
grams	grams	per cent	grams	per cent	grams	per cent
0.1	0.0035	3.5	0.0030	3.0	0.0016	1.6
0.1	.0002	0.2	.0038	3.8	.0010	1.0
0.5	.0043	0.9	.0104	2.1	.0003	0.1
0.5	.0186	3.7	.0067	1.3	.0011	0.2
1.5	.0078	0.5	.0063	0.4	.0036	0.2
1.5	.0136	0.9	.0100	0.7	.0021	0.1
2.5	.0167	0.7	.0210	0.8	.0015	0.1
2.5	.0133	0.5	.0320	1.3	.0049	0.2
3.5	.0192	0.5	.0379	1.1	.0058	0.2
3.5	.0156	0.4	.0176	0.5	.0032	0.1
5.0	.0113	0.2	.0217	0.4	.0041	0.1
5.0	.0151	0.3	.0218	0.4	.0043	0.1

a direct comparison between the various tube cleaning methods is available. Obviously, the residue was greatly decreased by more effective cleaning of the tube.

The material tabulated under "residue" in the "special cleaning" column includes the beads which floated on the meniscus, but since the total values are small the quantities held on the meniscus are believed to have been insignificant.

In at least 95 per cent of the analyses, the residue did not exceed 6 per cent of the total weight of beads in the sample, and generally the residue was about 2 per cent or less. For sample No. 20, analyzed by laboratory "C," a microscopic size analysis of the residue was made. The change in percentage finer values which would result if 6 per cent of this residue had been omitted from the analysis of sample No. 20 has been computed with the results shown in Table 13. This sample computation implies that no method

TABLE 13

CHANGE IN PER CENT FINER VALUES CAUSED BY OMITTING SIX PER CENT RESIDUE

Size	Base Sample		Residue		Portion Analyzed		
in microns	per cent finer	per cent of total	per cent of total	-6 per cent of total	per cent of original	per cent of new total	per cent finer
250	100.0						100.0
		10.3	0.0	0.0	10.30	11.0	
171	89.7						89.0
		17.7	11.0	-0.66	17.04	18.1	
124	72.0						70.9
		34.8	41.0	-2.46	32.34	34.4	
90	37.2						36.5
		22.2	38.0	-2.28	19.92	21.2	
64.7	15.0						15.3
		12.2	10.0	-0.60	11.60	12.3	
45.6	2.8						3.0
		2.8	0.0	0.0	2.80	3.0	
Totals		100.0		-6.00	94.00	100.0	

of cleaning the tube could reduce the residue sufficiently to cause a significant change in the analysis of the sample--the residue is not that important.

Other errors included in the results of these tests are large enough so that the effect of the cleaning method cannot be evaluated quantitatively within the scope of these tests. Sand grains show less tendency to adhere to the sides of the bottom withdrawal tube, and, therefore, analyses of sand samples would probably be less sensitive to differences in cleaning methods.

23. Accuracy of the Odén curve and tangent method--In an attempt to evaluate the accuracy of the Odén curve and tangent determinations of size distribution, three artificial sets of Odén curve coordinates were prepared for graphical analysis. A particular schedule of times of fall for the various sizes of particles was assumed. With these data, similar to actual laboratory results but free from the variations arising from laboratory data, repeated analyses were made to determine the consistency of results obtained by a single operator as well as the differences indicated by several operators. The three sets of data are indicated in Fig. 27.

The first set of data, Case I, defines the basic Odén curve of Fig. 15. This is a simple case of a smooth curve covering a size range of 20 to 350 microns. No inaccuracies of laboratory procedure are involved. The results should show maximum accuracy of analyses.

The second set of data, Case II, is composed of a typical set of points such as might be obtained from the laboratory for the sample corresponding to the basic curve of Case I. In this instance the points check the basic curve as well as generally found for this type of data as may be seen by comparing the points of Case II with the scatter of data in Fig. 15. The accuracy of analysis as shown by comparison with

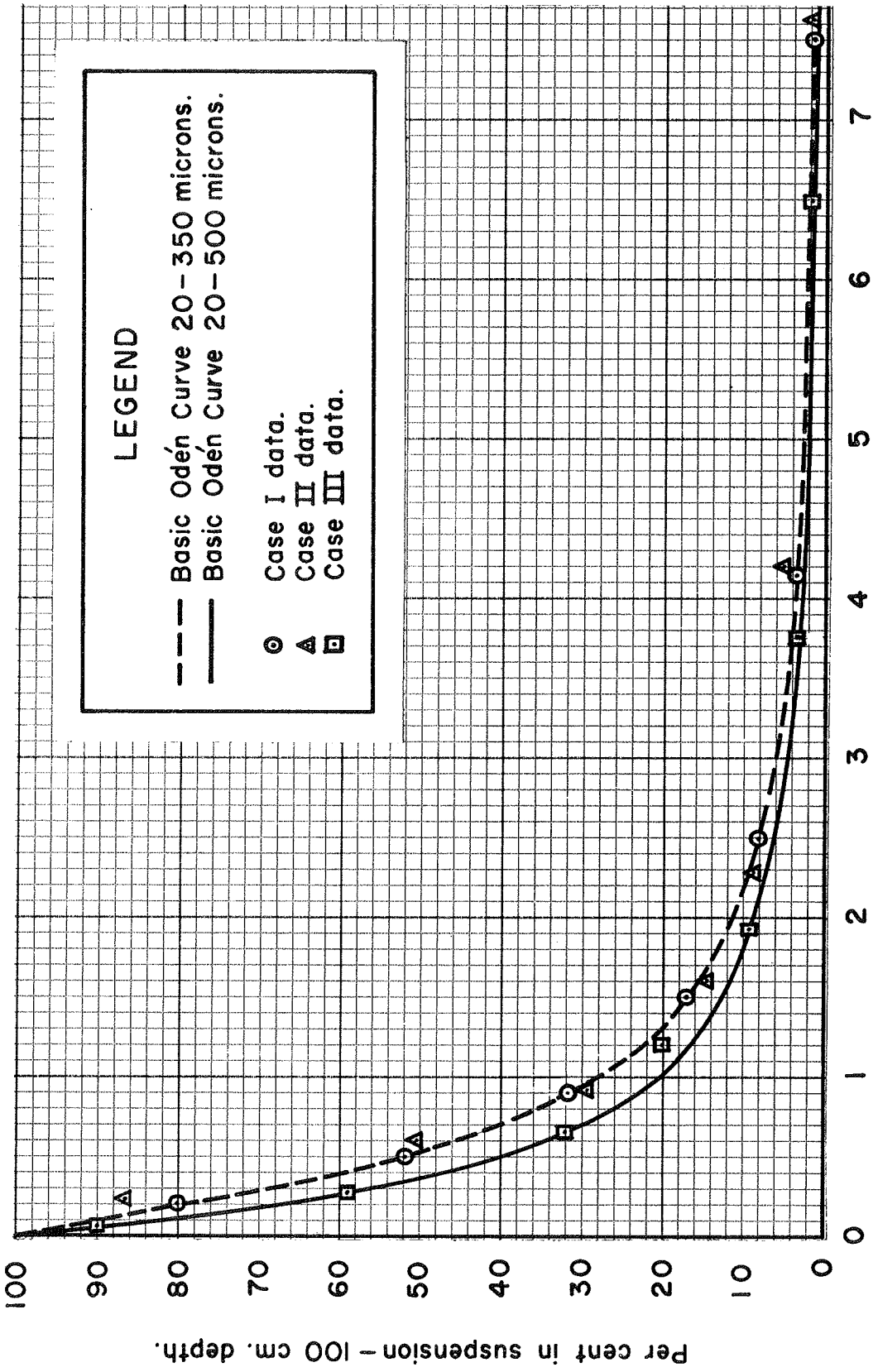


Fig. 27--Odén curves for graphical analysis study

the size distribution from which the basic Odén curve was computed, will reflect the effect of the scatter of the points about the basic curve.

The third set of data, Case III, coincides with the basic curve of Fig. 16, except for one point in the rapidly curving portion. At 1.2 minutes, on the time axis, the test data show 20 per cent remaining in suspension--3 per cent more than shown by the basic curve. It is possible for this problem to arise in the laboratory as a result of one of two conditions. Either a certain narrow size fraction is absent from the sediment sample, or an error in laboratory technique causes the particular withdrawal to appear light.

a. Comparison of individual analyses with average results

(1) A single operator, having above average knowledge of the character of the Odén curve and of the difficulties of the present method of graphical analysis, was able to repeat analyses of the typical set of coordinates of Case II ten times with a maximum deviation of +5 per cent, the greatest deviation otherwise being ± 3 per cent. "Per cent" as used in this relation refers to the difference between the per cent finer value of the individual analysis and that of the average. The variations in per cent finer values based on the average of all ten analyses are shown in Fig. 28a. The analyses were more consistent for the finer one-half of the sample.

(2) Nine operators representing three different laboratories independently analyzed three sets of Odén curve data by the curve and tangent procedure.

The study of Case I (Fig. 28c) showed a variation of ± 3 per cent from the average values of all analyses, if a few of the most erratic points are eliminated from consideration. The bulk of the variations are within ± 1 per cent for the finer half of the sample.

Case II is typical of the laboratory data of this investigation. As explained in Section 19, the results of the first withdrawal are sometimes so undependable as to be almost useless. Therefore, in drawing the Odén curve, the first withdrawal point may have to be disregarded with corresponding loss of definition of the curve. Fig. 28b shows that the

personnel involved handled this difficult problem in much the same manner. Here, as in the previous cases, the spread is most pronounced as the sharply breaking part of the curve is entered, and reduces to ±1 per cent as the slope of the curve approaches zero. An over-all variation of ±3 per cent describes the accuracy of the graphical analysis, although several points are outside these values.

An example which may well be more normal than extreme is the condition of Case III in which one withdrawal may be lighter or heavier than expected for a smooth curve, resulting in a pair of coordinates not consistent with the remainder of the curve. If for instance, a certain size fraction is nearly lacking in an otherwise well-graded material, the Odén curve will contain a straight line segment in the neighborhood of the time for this particular size to fall 100 cm. An additional problem has been introduced in Case III, for the maximum size of particle in the sample has been increased to 500 microns and the basic curve as shown in Fig. 27 is steeper than in the previous cases.

The analyses are plotted in Fig. 28d and show a variation of 4 per cent from the average. In contrast to cases previously studied, the spread remains fairly constant throughout the size distribution, showing that the displaced point had a definite effect upon the interpretation of the shape of the entire curve.

b. Individual deviations from the basic distribution--The data of Case I define the basic Odén curve for size range 20 to 350 microns, for which the size frequency distribution is shown in Table 4. The analysis by the curve and tangent method can be carried a step further in this instance to point up the deviations from the known size distribution, as distinguished from previous deductions which have been confined to the scatter about the average cumulative size distribution as found by graphical analysis. These data are shown in Fig. 28e. While most of the determinations by the curve and tangent method are within 4 per cent, some points determined by experienced operators are in error by 7 and 8 per cent. Errors of at least 7 per cent must be expected in the curve and tangent method even under the optimum conditions of this test of experienced analysts, well defined Odén curve, and uniform size distribution.

c. Comparison of average analyses with basic size distribution--Although the Odén curve information employed in the graphical investigation was not the product of an actual size analysis with the bottom withdrawal tube, yet the three cases closely reflect the accuracy of laboratory technique. The variations between the average results obtained by the several operators and the basic distributions indicate the effect of the shape of the settlement curve on size distribution.

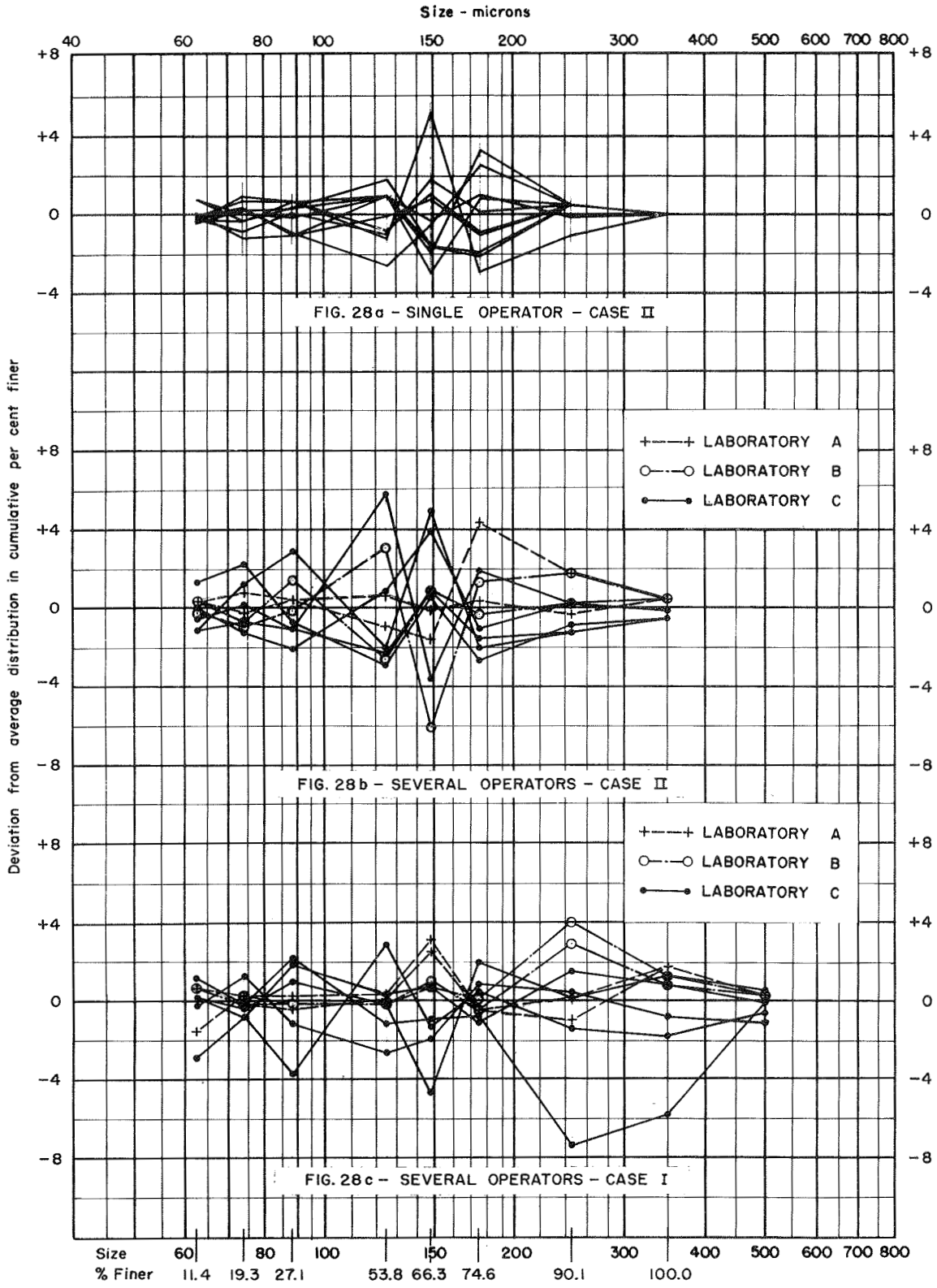


Fig. 28--Consistency of Odén

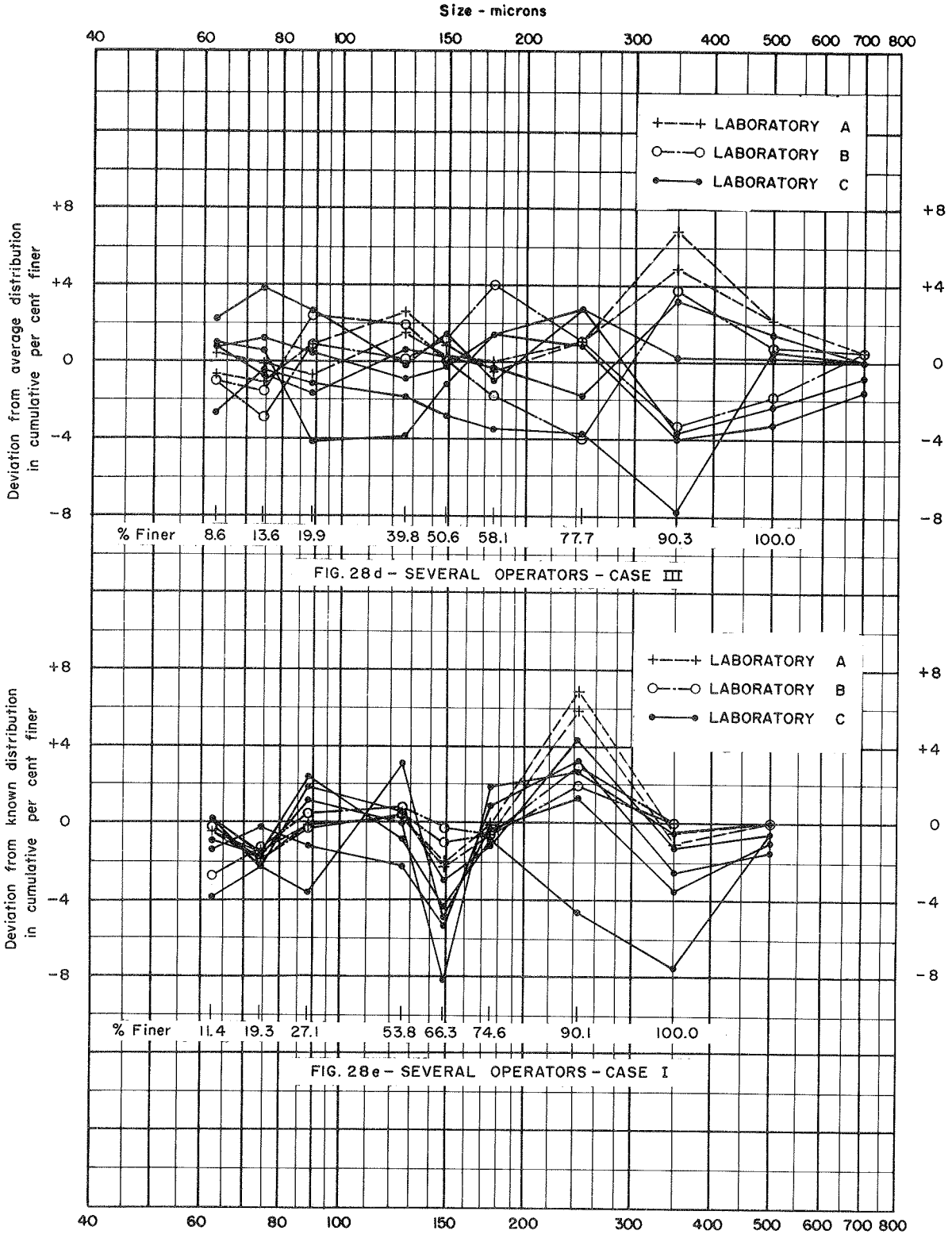


FIG. 28d – SEVERAL OPERATORS – CASE III

FIG. 28e – SEVERAL OPERATORS – CASE I

curve and tangent results

Several operators analyzing Case I, which is identical with the basic curve of Fig. 15, determined an average distribution only slightly different from the base distribution. Fig. 29a shows that the differences reach 3.5 per cent, although they are generally much smaller. The tendency is to show some material coarser than actually contained in the base sample, but the quantities involved are rather small.

One operator, repeating his own analysis of Case II ten times, arrived at an average distribution with a slightly larger maximum error than that of the several operators. Both average curves are compared with the basic curve in Fig. 29b. The shape of the average Odén curve is sufficiently different from that of the basic curve to cause a difference of nearly 20 per cent at 175 microns. Then, too, many operators indicated one per cent or less of material coarser than 350 microns in the sample but this amount is considered insignificant.

In the third instance, while the artificial Odén curve contains a displaced point within the sharply breaking portion, the average distribution differs less than 5 per cent (Fig. 29a) from that given by the basic curve. Assuming that this displaced point is in error by the amount of the displacement from the curve, the differences obtained by the analyses are a valid expression of errors. If the displacement of the point is the result of a lack of a given size range in the sample, then the magnitude of the errors at 149 and 125 microns is unknown and that at 175 microns is only approximate.

The analyses fail to define the upper limits of size present in the samples. For Case III, the average distribution showed that over 2 per cent of the sample was coarser than 500 microns. The different laboratories are identified with different types of interpretation of the coarser portions of the Odén curve.

24. Maximum concentration for bottom withdrawal tube analyses--None of the glass bead samples used in the experiments on the accuracy of the bottom withdrawal tube method contained more than 22 per cent of silt sizes, and these were almost all in the coarse silt range. For samples of this type and size range, there is no indication that 10,000 p.p.m. is too high a concentration for accurate size analysis.

There is an inherent difficulty in the analysis of samples containing natural sediments in the clay and silt sizes, and this necessitates

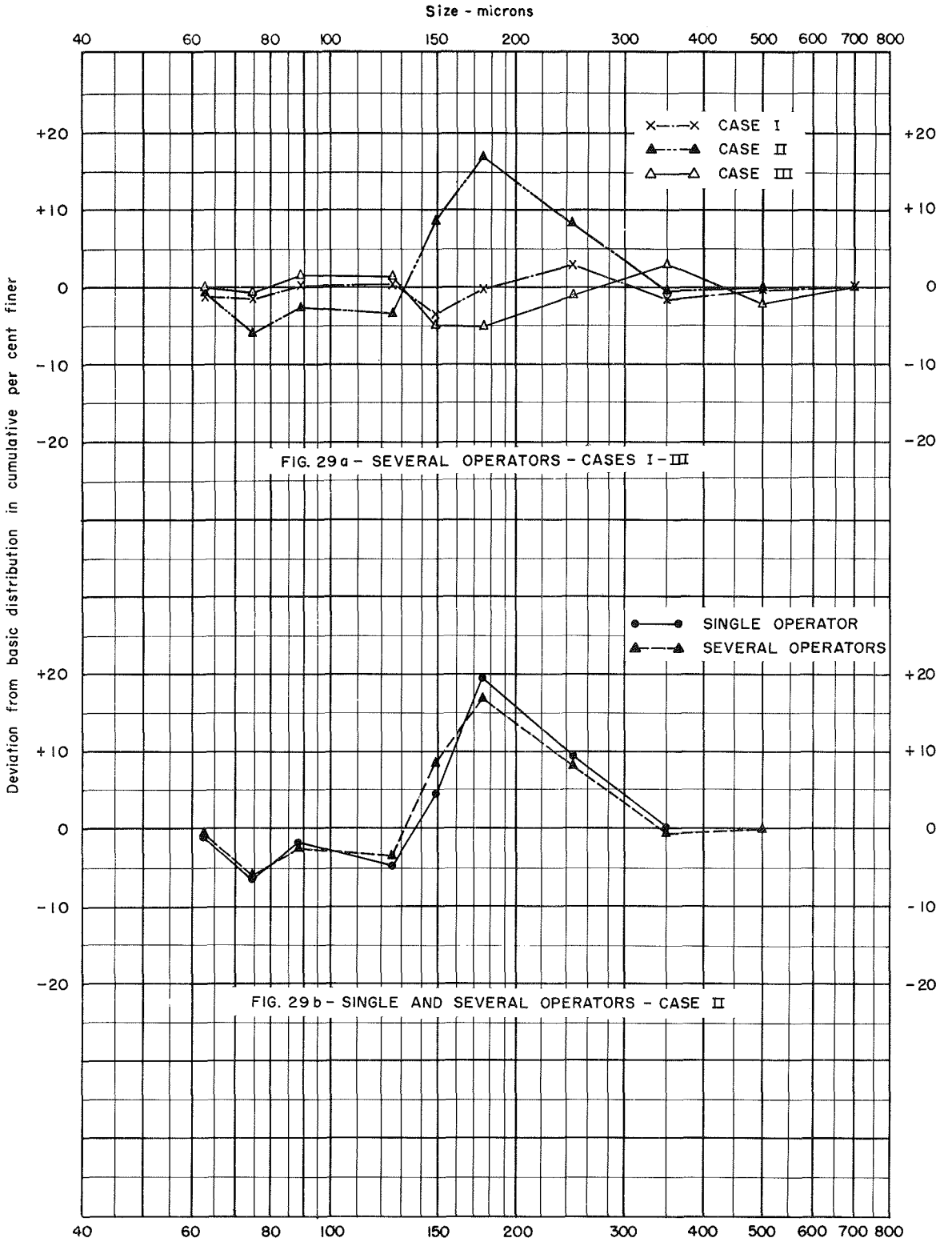


Fig. 29--Comparison of average analyses with basic distributions

limiting the maximum concentrations to values far below 10,000 p.p.m. Flocculation, the combining of individual particles in suspension into larger bodies of loosely compacted grains, creates masses of sediment which fall faster than indicated by the fall velocities of the individual component particles, and distorts the natural shape of the velocity distribution curve. Flocculation may be reduced by using distilled water as a suspension medium, or by the addition of a chemical dispersing agent, but in any event the analyses must be made with low concentrations of sediment. Undoubtedly the maximum concentration for satisfactory analysis varies greatly from sample to sample. However, experience in field laboratories has indicated that the concentration in a dispersed suspension should not exceed 3,500 p.p.m., without investigation of the probable errors introduced by use of a higher concentration. For example, a sample of 10,000 p.p.m. could be split and one analysis made on a concentration of 5,000 p.p.m. and another on a concentration of 2,500 p.p.m. and the results compared to see whether, for the given type of sample, the 5,000 p.p.m. is excessive or not.

V. CONCLUSIONS

25. Conclusions--The investigation of the accuracy of the bottom withdrawal tube method of size analysis, on the basis of analyses of glass bead samples, in the range of sand sizes up to 700 microns has led to the following conclusions:

a. Glass bead samples--The glass bead samples provide a valuable means of checking physical size against fall velocity. Errors of 4 or 5 per cent may be possible in the base samples composed of these glass spheres, but the average error is undoubtedly much less than these amounts. These errors are considered insignificant in terms of application to final results.

b. Accuracy of the Odén curve and tangent method--The accuracy with which the laboratory data checked the basic Odén curves of Figs. 13 to 17, indicates that for concentrations of 3,000 p.p.m. to 10,000 p.p.m. the analyses should be well within 5 per cent, with the possible exception of results for the coarsest sizes. However, the final results of the analyses as plotted in Figs. 18 to 23 show many variations greater than 10 per cent and several over 20 per cent. The per cent figures used here are the differences between the per cent finer values for the base sample and those of the analysis. This increase in errors is the result of the use of the curve and tangent method.

By the nature of the curve and tangent method, one would expect that the errors developed therein would be sometimes positive, and sometimes negative, and would tend to counterbalance over a group of several analyses. The average error for a group of analyses might be considered fairly independent of the curve and tangent errors, which suggests that the average errors of a group of analyses indicate the accuracy of the data as obtained from the laboratory. This implies that the average errors of Fig. 24 reflect the accuracy of the laboratory data, while the more erratic nature of the individual analyses as shown by Tables 7 and 8, results from the curve and tangent interpretation of the laboratory data.

The independent study of the curve and tangent method, presented in Section 23, also illustrates the magnitude of the errors and the relative consistency of the results of the tangent method. Under the most favorable conditions to be found in samples of sand sizes: uniform size distribution, no coarse sand particles, and a well defined Odén curve, and considering that all points defining the curve are perfectly determined, one must still expect errors

of 7 per cent to occur frequently at points in an analysis. When the errors commonly found in the laboratory data are combined with those inherent in the tangent method, the combination produces frequent errors of 10 per cent and some errors of about 20 per cent. As long as the curve and tangent method is used as a part of the bottom withdrawal tube analysis, the results of individual analyses can never be precise, and errors of the magnitude of those shown in Tables 7, 8, 10, and 11 must be expected.

Presumably the skill of the analyst is an important factor in the results of the curve and tangent method; however, the tests were not sufficiently detailed to define that point. Only experienced operators performed these tests.

c. Accuracy of laboratory technique--These tests showed that there are three weaknesses in the laboratory method or technique: (1) At concentrations of 200 p.p.m. the analyses are much less accurate than at the higher concentrations, and as indicated in Table 10, the effect of the low concentrations is mainly to increase the errors in the smaller particle sizes. Extremely small quantities of sedimented material were found in the withdrawals which determine the part of the Odén curve used for the smaller particle sizes and from these portions of the curve the largest errors were derived. As pointed out in Section 19, the sum of the weights of the withdrawals was frequently greater than that of the original sample. Errors in weighing, or contamination of these very small withdrawals, developed most of the +5.2 per cent error in the per cent finer values shown by the average results for samples with a concentration of 200 p.p.m. (2) The results of the first withdrawal tended to be erratic, and the larger the size of particles contained in a sample, the less dependable were the results of the first withdrawal. When particles over 246 microns (1/4 mm.) are contained in the sample, the present technique fails to provide adequate data to define that portion of the Odén curve on which the analysis of the coarser material is based. (3) There was some tendency for the glass beads to adhere to the sides and shoulders of the bottom withdrawal tube, and some beads floated on the meniscus. Most of this difficulty may be eliminated by using the type of cleaning of the tube which is described in Section 22. For the samples in the size range 20 to 700 microns, there is a direct comparison of one set of analyses made with the more effective type of cleaning, one made with "usual" types of cleaning, and one made with a combination of methods. In this size group the maximum amount of material which adhered to the tube was about 4 per cent, and the amount was generally less than 1 per cent of the total sample. In samples covering a smaller size range, the percentage of material adhering to the tube was sometimes around 6 per cent but was generally much less. Even with 6 per cent, the effect of omitting this residue as shown by a computed example in Section 22, is still less than 1 per cent on the basis of the total sample. The effect of the method of cleaning is apparently

too small to define by this series of tests. When analyzing sand samples, the effect of the method of cleaning would probably be even less important.

For concentrations of 3,000 p.p.m. and over, the laboratory data check the computed Odén curves of Figs. 13 to 17 within five per cent except for some of the first and second withdrawals. If the average errors of analysis as shown in the table in Fig. 24 are also considered representative of the accuracy of the laboratory data, then for all of the higher concentrations the laboratory data are fairly satisfactory. These considerations support the theoretical soundness of the method of analysis. They also indicate that laboratory techniques such as dispersion of material in the tube, timing of withdrawals, and weighing and handling procedures are in general relatively satisfactory. Unfortunately serious difficulties are encountered at larger grain sizes, at low concentrations, and with variations from sample to sample and from point to point within a sample.

d. Effect of concentration on analytical results--At concentrations of 200 p.p.m., the results of the bottom withdrawal tube analyses are not very satisfactory, although the average error is only approximately 5 per cent. At concentrations of 1,000 and 3,000 p.p.m., the average error is 2.5 to 2.9 per cent, and the maximum error in an individual analysis is under 10 per cent about half of the time. At concentrations of 5,000 to 10,000 p.p.m., the average error is about 1 per cent, and the maximum error in an individual analysis is less than 10 per cent for about 70 per cent of the samples. For the sand sizes of these tests there is a definite trend toward increasing accuracy as the concentration in the sample increases. See Tables 7, 8, 10, and 11, and Fig. 24. The average errors shown here in "per cent" are the average differences between the per cent finer values for the base sample and those for the analysis of the sample.

e. Effect of particle size on analytical results--Table 8 and Fig. 24 indicate no definite change in accuracy of analysis as the size of sediment involved changes. Tables 7 and 11 show that the results of the analyses of individual samples and of portions of samples become somewhat more erratic as the size of sediment contained in the sample increases. The initial withdrawal as found in Figs. 13 to 17, is never very accurate and becomes increasingly inaccurate as the maximum size of sediment increases from 350 microns upward. Even the second withdrawal is sometimes undependable. Because the first withdrawal was largely disregarded in samples of 350 microns maximum size and larger, the individual results in the larger sand sizes were relatively more erratic, but in spite of the lack of definition of the Odén curve at this point, the results as shown by average errors are still about as accurate as for the smaller sizes. These tests were made on samples having smooth and regular size distributions. For samples

having irregular distributions of coarse sizes, the lack of definition of the Odén curve might cause more serious errors.

f. Evaluation of the bottom withdrawal tube method--The degree with which the accuracy of the bottom withdrawal tube method satisfies the needs of any size analysis program, depends upon the requirements of the program. The best evaluation of the method would usually require an individual study of these tests in terms of the essential needs of a given size analysis problem. However, it is possible to consider the accuracy of the test results from three general viewpoints: (1) There are instances in which the quantity of sand of a given size must be known for use in problems of transport or deposition. Tables 10 and 11 should be studied in this connection. In very general terms, an individual analysis showed the proper quantity of a sand fraction, or size class, within 10 per cent for a third of the time, within 20 per cent half of the time, within 40 per cent three quarters of the time, and within 80 per cent all but one-sixteenth of the time. But if the coarsest fraction contained 10 per cent of the total sample, the determination of that sand fraction was 100 per cent in error about one time in six. This accuracy is generally unsatisfactory for the determination of material within individual size classes. The accuracy could be improved by making several analyses and using the average values. (2) For some purposes, such as determination of median grain size, the per cent finer curve for the sample is needed. The differences between the per cent finer distributions for the sample and those for the individual analyses are shown in Figs. 18 to 23, and average results in Fig. 24. The results of individual analyses are quite erratic, as may be seen in Table 8, and the maximum errors found in the samples are frequently large. For most of the results of Figs. 18 to 23 the errors vary from plus to minus, so that the per cent finer curves viewed as a whole are much more consistent than the errors contained in individual parts of the curves would indicate. Single per cent finer distributions cannot be considered precise. Whether the accuracy is satisfactory would depend upon the purpose for which the curves are to be used. (3) For evaluating the results of a general size program, the accuracy of the average results obtained by the bottom withdrawal tube method might be the controlling consideration. If a program is comprehensive enough in time or in thoroughness of coverage with size analyses, so that the results depend upon the average of many bottom withdrawal tube analyses, then there is every expectation that the over-all results will be highly accurate. Due to the errors in the curve and tangent method individual analyses are erratic, but average results largely eliminate these errors.

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